

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040
Circuit Analysis

November 3, 2000

Problem Set #10–Solutions

Problem 10.1: The circuit in Figure 1 is at initial rest.

- (a) Find the system function $H(s)$ that relates the output $I_{out}(s)$ to the input $V_{in}(s)$.
- (b) Find the impulse response of the system $h(t)$.

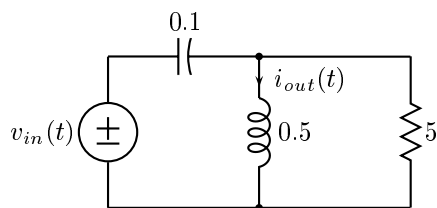


Figure 1: Circuit for Problem 10.1.

Solution:

- (a) Let $I_{in}(s)$ denote the current flowing out of the voltage source. Then we can use a current divider to relate $I_{in}(s)$ to $I_{out}(s)$.

$$I_{out}(s) = I_{in}(s) \cdot \frac{5}{\frac{s}{2} + 5} = I_{in}(s) \cdot \frac{10}{s + 10}$$

To determine $I_{in}(s)$ we can determine the equivalent impedance of the combined elements, since

$$I_{in}(s) = \frac{V_{in}(s)}{Z_{eq}(s)}.$$

But,

$$\begin{aligned} Z_{eq}(s) &= \frac{10}{s} + \frac{\frac{s}{2} \cdot 5}{\frac{s}{2} + 5} = \frac{10}{s} + \frac{5s}{s + 10} \\ &= \frac{5(s^2 + 2s + 20)}{s(s + 10)} \end{aligned}$$

Therefore,

$$I_{out}(s) = V_{in}(s) \cdot \frac{10}{s + 10} \cdot \frac{s(s + 10)}{5(s^2 + 2s + 20)} = \frac{2s}{s^2 + 2s + 20}$$

and

$$H(s) = \frac{2s}{s^2 + 2s + 20}$$

(b) We can manipulate $H(s)$ into the form

$$H(s) = \frac{2(s+1)}{(s+1)^2 + 19} - \frac{\frac{2}{\sqrt{19}} \cdot \sqrt{19}}{(s+1)^2 + 19}$$

from which we see that

$$h(t) = 2e^{-t} \cos \sqrt{19}t - \frac{2}{\sqrt{19}}e^{-t} \sin \sqrt{19}t.$$

Problem 10.2: A parallel RC circuit is connected in series with an unknown circuit N excited by a current source as shown in Figure 2(a). The current source waveform is shown in Figure 2(b). The goal of the problem is to design the network N so that the voltage $v(t)$ will be that shown in Figure 2(c).

- Use Laplace transform methods to determine the voltage $v_2(t)$.
- Sketch $v_1(t)$ for $t \geq 0$.
- Determine the impedance $Z_1(s)$ of the circuit N .
- Determine the circuit N .

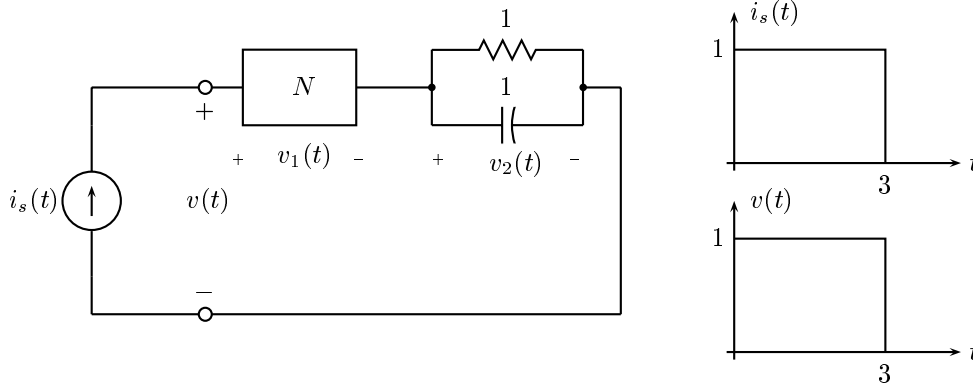
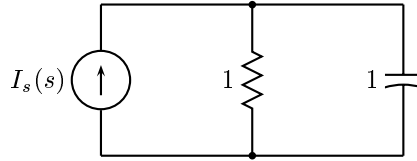


Figure 2: Circuit for Problem 10.2 with the current excitation and voltage response shown. The voltage $v_1(t)$ and the circuit N are to be determined.

Solution:

- Notice that the whole current $i_s(t)$ passes through the parallel connection. Therefore, we can find $v_2(t)$ by finding the voltage drop in the following circuit.



Let the voltage drop across the capacitor be $s(t)$, when $i_s(t) = u(t)$. Then

$$S(s) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} \cdot \frac{1}{s} = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

and

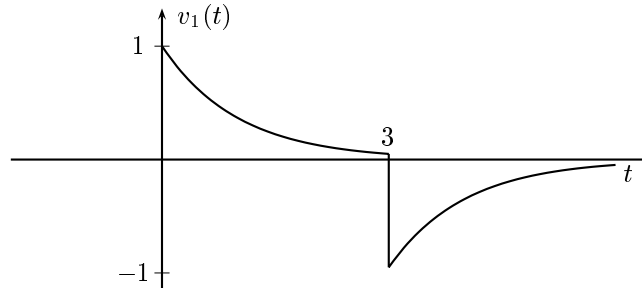
$$s(t) = [1 - e^{-t}]u(t).$$

Therefore,

$$\begin{aligned} v_2(t) &= s(t) - s(t-3) = [1 - e^{-t}]u(t) - [1 - e^{t-3}]u(t-3) \\ &= \begin{cases} 0, & t < 0 \\ 1 - e^{-t}, & 0 \leq t \leq 3 \\ (e^3 - 1)e^{-t}, & 3 < t \end{cases} \end{aligned}$$

(b) $v_1(t) = v(t) - v_2(t)$. Therefore

$$v_1(t) = \begin{cases} 0, & t < 0 \\ e^{-t}, & 0 < t < 3 \\ -(e^3 - 1)e^{-t}, & 3 < t. \end{cases}$$



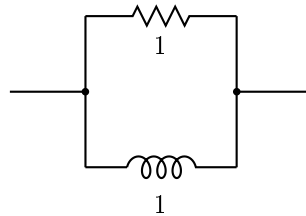
(c) The impedance of the overall system should have a constant value of 1. Therefore, if $Z_2(s)$ denotes the impedance of the parallel RL

$$Z_1(s) + Z_2(s) = 1.$$

This means that

$$Z_1(s) = 1 - Z_2(s) = 1 - \frac{1}{s+1} = \frac{s}{s+1}.$$

(d) One circuit that will have this impedance is a parallel RL .



Problem 10.3: An oscillator is a circuit whose system function has a pair of complex conjugate poles located $s = \pm\omega_0$, where ω_0 is the frequency of oscillation. Once started it will continue to produce a sinusoidal output $v_{out}(t)$. In the network in Figure 3 determine the value of K for which the system will oscillate and determine the frequency of oscillation.

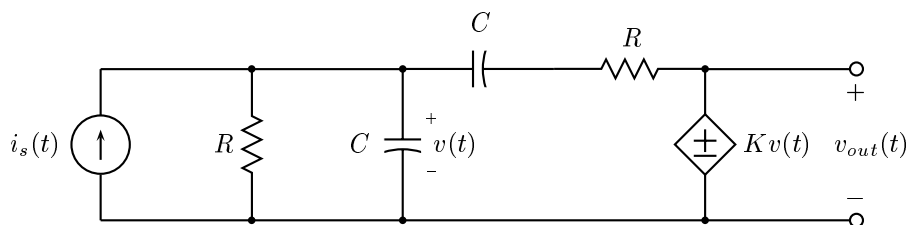
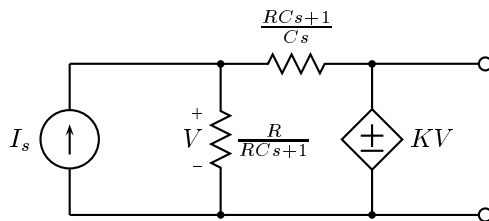


Figure 3: An oscillator circuit for Problem 10.3.

Solution: To begin we can simplify the circuit in the Laplace domain by replacing the elements in series and in parallel by their equivalent impedances. This gives the circuit shown below.



Since $V_{out} = KV$, this is not included on the figure. Let the mesh current in the right mesh be $I(s)$ (and the mesh current in the left mesh be $I_s(s)$). Then

$$(I(s) - I_s(s)) \frac{R}{RCs + 1} + I(s) \left(\frac{RCs + 1}{Cs} \right) - K(I(s) - I_s(s)) \frac{R}{RCs + 1} = 0$$

We can solve this equation for the system function $H(s) = \frac{I(s)}{I_s(s)}$.

$$H(s) = \frac{(1 - K)RCs}{(1 - K)RCs + R^2C^2s^2 + 2RCs + 1} = \frac{\frac{1-K}{RC}s}{s^2 + \frac{3-K}{RC}s + \frac{1}{R^2C^2}}$$

From this we see that the poles will lie on the imaginary axis when $K = 3$. When $K = 3$, the poles lie at $\pm \frac{1}{RC}$, so the frequency of oscillation is $\omega_0 = \frac{1}{RC}$.

Problem 10.4: For the two-terminal pair circuit shown in Figure 4 the relation between $v(t)$ and $i(t)$ is

$$v(t) = i(t) + 3 \frac{di(t)}{dt}$$

when the terminals b and b' are open-circuited and

$$v(t) = \frac{di(t)}{dt}$$

when the terminals b and b' are short-circuited. Determine a possible circuit having these properties.

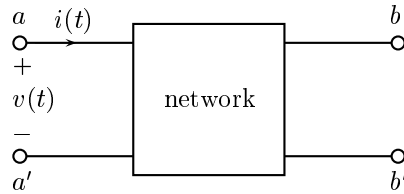


Figure 4: A two-terminal network to be designed in Problem 10.4.

Solution: The answer to this problem is not unique. Here is one possibility

