GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

November 17, 2000

Problem Set #12–Solutions

Problem 12.1: Find $v_{out}(t)$ if $v_{in}(t) = 3 + 4\sin(1000t)$. The terminals are *open-circuited.*

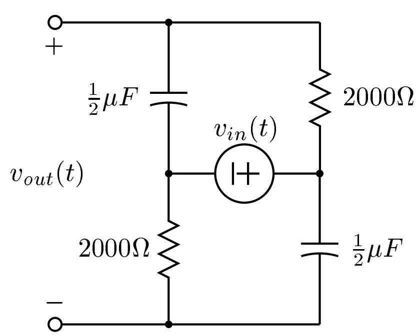
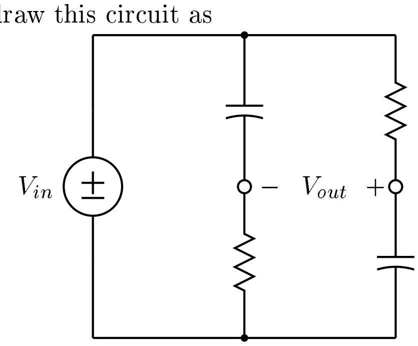


Figure 1: Circuit for Problem 12.1.

Solution: We can redraw this circuit as



Then

$$
\begin{aligned}
V_{out} &= \left(\frac{\frac{1}{jC\omega}}{\frac{1}{jC\omega} + R} - \frac{R}{\frac{1}{jC\omega} + R} \right) V_{in} \\
&= \frac{1 - jRC\omega}{1 + jRC\omega} V_{in}
\end{aligned}
$$

Now, we observe

$$
\begin{aligned}
\omega = 0 : &\quad V_{out} = V_{in} \\
\omega = 1000 : &\quad V_{out} = \frac{1-j}{1+j} V_{in} = e^{-j\frac{\pi}{2}} V_{in}
\end{aligned}
$$

Therefore,

$$v_{out}(t) = 3 + 4\sin(1000t - \frac{\pi}{2}) = 3 - 4\cos(1000t).$$

Problem 12.2: Determine $i(t)$ for all t for the circuit in Figure 2.

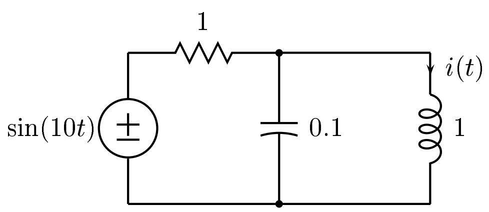


Figure 2: Circuit for Problem 12.2.

Solution:

$$H(j\omega) = \frac{I}{V_s} = \frac{1}{j\omega} \cdot \frac{\frac{10}{\frac{10}{j\omega}+j\omega}}{\frac{10}{\frac{10}{j\omega}+j\omega}+1} = \frac{10}{10-\omega^2+j10\omega}$$

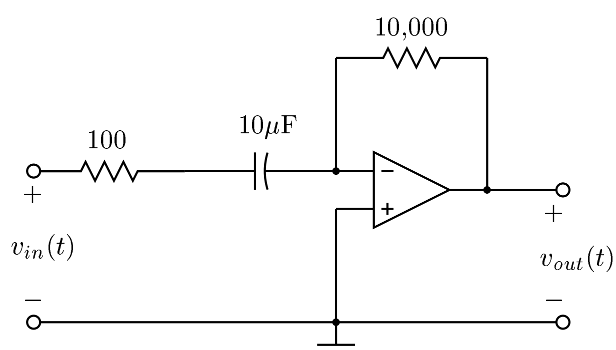
Letting $\omega = 10$ gives

$$H(j10) = \frac{10}{10-100+j100} = \frac{1}{\sqrt{181}}e^{j\tan^{-1}(\frac{10}{9})-j\pi}$$

Therefore,

$$i(t) = \frac{1}{\sqrt{181}}\sin(10t + \tan^{-1}(\frac{10}{9})) = 0.074\sin(10t + \tan^{-1}(\frac{10}{9}))$$

Problem 12.3:



(a) Calculate the frequency response of the above circuit.

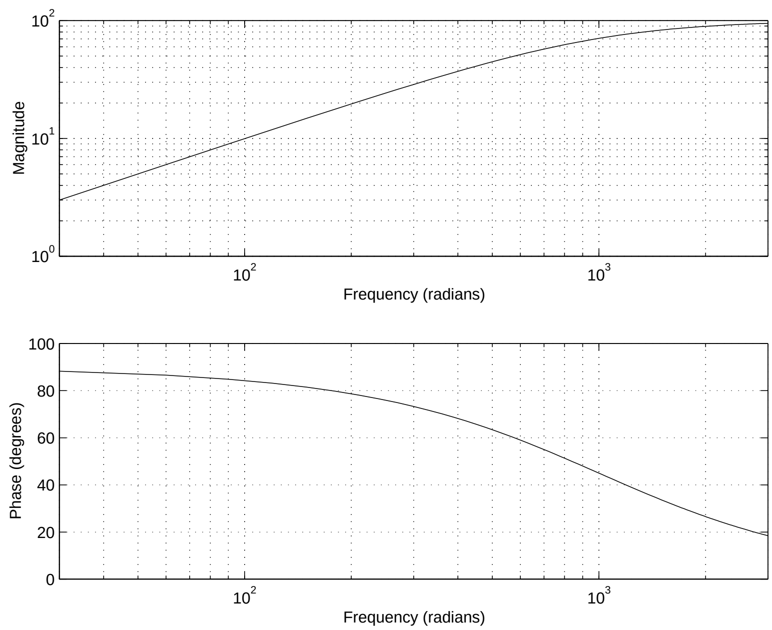
(b) Plot the magnitude response.

(c) Plot the phase response.

Solution:

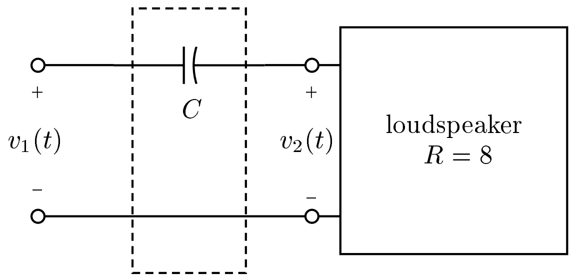
(a)

$$H(j\omega) = -\frac{Z_f(j\omega)}{Z_i(j\omega)} = -\frac{10^4}{100 + \frac{1}{j10^{-5}\omega}}$$
$$= -\frac{j(0.1)\omega}{j0.001\omega + 1}$$



(b,c)

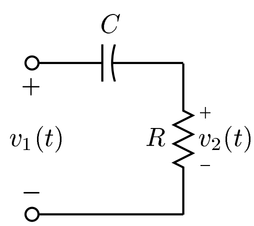
Problem 12.4: Many loudspeaker systems consist of two loudspeakers: the woofer, which reproduces the low frequency part of the signal, and the tweeter, which reproduces the high frequency part of the signal. A crossover network is used to select the high frequency part of the signal and feed it into the tweeter. Such a network functions as a highpass filter. The entire audio signal is applied at the terminals $a - a'$.



(a) Assuming that the equivalent circuit for the tweeter consists of just a resistor with a resistance of R, plot the pole-zero pattern of the system function that relates $v_2(t)$ to $v_1(t)$ and sketch the frequency response curves (magnitude and angle).

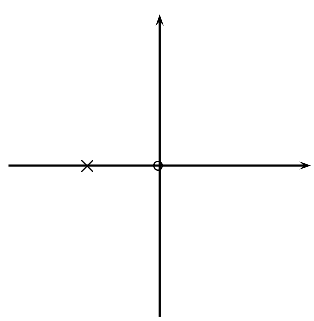
(b) If $R = 8\Omega$, find the value of the capacitance C so that the half-power frequency of the highpass filter is 5 kHz ($= 2\pi(5000)$ rad/s).

Solution:

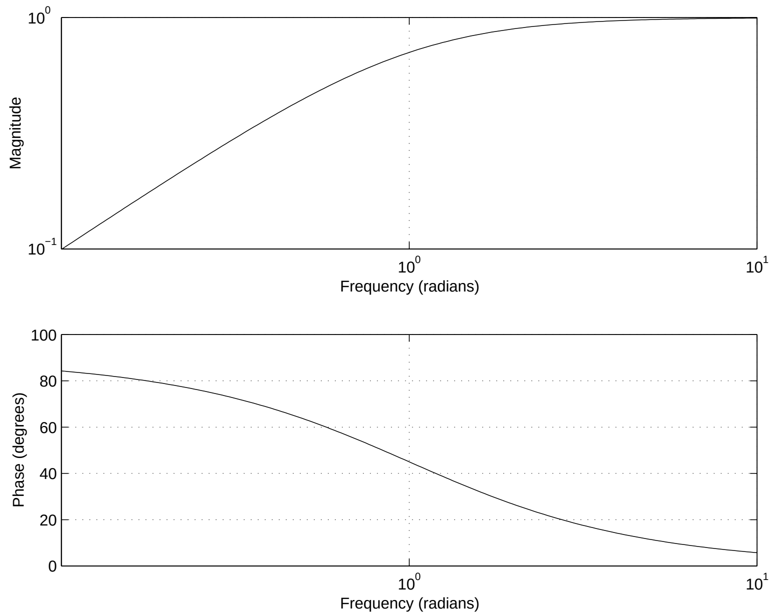


$$H(s) = \frac{R}{R + \frac{1}{Cs}} = \frac{s}{s + \frac{1}{RC}}$$

(a)



Using the MATLAB function `freqs` again gives the following plot (for positive frequencies) if $RC = 1$.



(b)

$$\frac{1}{RC} = 2\pi(5000)$$

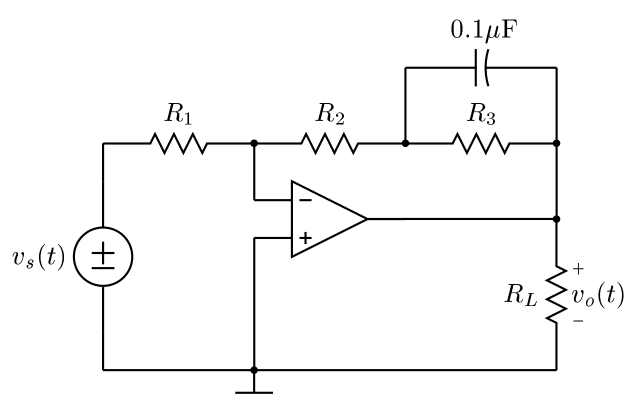
Thus,

$$C = \frac{1}{80,000\pi} \approx 4\mu F$$

Problem 12.5: In the circuit below the value of R_1 is 10kΩ.

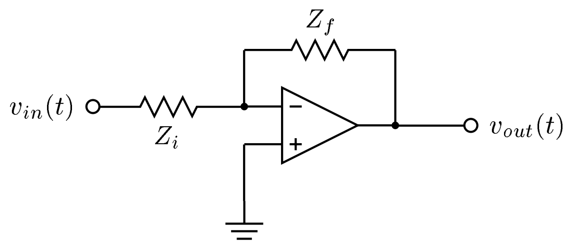
(a) Determine the values of R_2 and R_3 so that the gain (magnitude of the frequency response) at low frequencies is 5 and the gain at high frequencies is 2.

(b) Determine the frequency at which the gain is midway between these two values, i.e. the frequency at which the gain is 3.5.



Solution:

(a) This circuit is a special case of an inverting amplifier as shown below.



The frequency response of such a system is known to be

$$H(j\omega) = -\frac{Z_f(j\omega)}{Z_i(j\omega)}.$$

For this circuit, the input impedance is simply

$$Z_i = R_1$$

and the other three elements contribute to the feedback impedance

$$\begin{aligned} Z_f &= R_2 + \frac{R_3 \frac{1}{j\omega C}}{R_3 + \frac{1}{j\omega C}} = R_2 + \frac{R_3}{1 + j\omega R_3 C} \\ &= \frac{(R_2 + R_3) + j\omega R_2 R_3 C}{1 + j\omega R_3 C}. \end{aligned}$$

Therefore,

$$H(j\omega) = -\frac{(R_2 + R_3) + j\omega R_2 R_3 C}{R_1(1 + j\omega R_3 C)}.$$

To measure the low frequency gain, we can let $\omega = 0$ and take the magnitude

$$G_{LF} = \frac{R_2 + R_3}{R_1}.$$

To measure the high frequency gain, we can take the (magnitude of the) limit as ω grows large

$$G_{HF} = \frac{R_2}{R_1}.$$

Since $R_1 = 10\text{k}\Omega$ and we want $G_{LF} = 5$ and $G_{HF} = 2$, we can substitute and solve for R_2 and R_3.

$$\begin{aligned} 5 &= \frac{R_2 + R_3}{10k\Omega} \Longrightarrow R_2 + R_3 = 50k\Omega \\ 2 &= \frac{R_2}{10k\Omega} \Longrightarrow R_2 = 20k\Omega \end{aligned}$$

Therefore, we want to choose $R_2 = 20k\Omega$ and $R_3 = 30k\Omega$.

(b) With the values substituted, the frequency response is

$$H(j\omega) = -\frac{5 + j\omega(0.6)}{1 + j\omega(0.3)}.$$

To find the value of ω for which the magnitude is 3.5, we solve the equation

$$\frac{25 + \omega^2(0.6)^2}{1 + \omega^2(0.3)^2} = (3.5)^2.$$

This gives $\omega = 4.14$.
