GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

September 1, 2000

Problem Set #1 – Solutions

Problem 1.1: The voltage waveform for the voltage source in the network of Figure 1 is

$$v_s(t) = \begin{cases} \sin 2\pi(100)t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

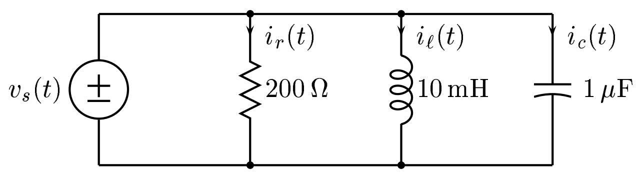


Figure 1: Circuit for Problem 1.1.

(a) Determine $i_r(t)$.

(b) Assuming $i_\ell(0) = 0$, determine $i_\ell(t)$.

(c) Determine $i_c(t)$.

Solution:

(a) The voltage drop across all three elements is $v_s(t)$. Thus, we can find their currents using a simple application of the $v - i$ relations. For the resistor the current is proportional to the voltage

$$i_r(t) = \frac{1}{200} v_s(t) = \begin{cases} 0.005 \sin 2\pi(100)t, & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

(b) The current through an inductor is proportional to the integral of the voltage drop across its terminals. Therefore, we can write

$$i_\ell(t) = \frac{1}{.01} \int_0^t v_s(\beta)\, d\beta + i_\ell(0) = 100 \int_0^t v_s(\beta)\, d\beta + i_\ell(0).$$

Assuming that the initial value of the inductor current is zero, this gives

$$i_\ell(t) = \begin{cases} \frac{1}{2\pi} \left[1 - \cos 2\pi(100)t\right], & t \geq 0 \\ 0, & t < 0. \end{cases}$$

To determine the value of $i_\ell(t)$ for $t < 0$, we appealed to the fact that the integral is equal to the area under the curve. Since $i_\ell(t) = 0$, for $t < 0$, the integral under this portion of the curve is zero also.

(c) The current through a capacitor is proportional to the derivative of the voltage drop across its terminals. Therefore,

$$i_c(t) = C\frac{dv_s(t)}{dt} = \begin{cases} 10^{-6} \cdot 2\pi(100)\cos 2\pi(100)t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$= \begin{cases} 2\pi 10^{-4}\cos 2\pi(100)t, & t \geq 0 \\ 0, & t < 0. \end{cases}$$

Problem 1.2: For the circuit in Figure 2 write a sufficient set of KCL equations in terms of the voltage variables that are labeled. You should incorporate Ohm's Law for the resistors. Exploit any obvious series and parallel connections to reduce the number of equations and variables.

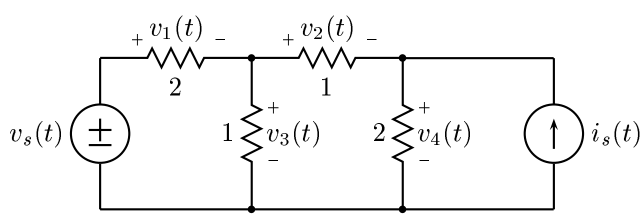
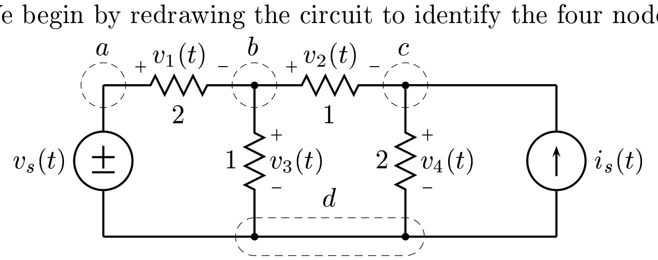


Figure 2: Circuit for Problem 1.2.

Solution: We begin by redrawing the circuit to identify the four nodes.



Since the circuit contains four nodes, it will be sufficient to write only three KCL equations. The equation at node a, will tell us only that the current flowing upwards through the voltage source is the same as $i_1(t)$, since these two "elements" are connected in series; we can ignore this one also. That leave only nodes b and c.

$$\text{node } b: \quad \frac{v_1(t)}{2} - v_2(t) - v_3(t) = 0$$

$$\text{node } c: \quad v_2(t) - \frac{v_4(t)}{2} = -i_s(t)$$

Problem 1.3: For the circuit in Figure 3 write a sufficient set of KVL equations to constrain all of the element variables in terms of the current variables that are indicated. You should incorporate Ohm's Law for the resistors. Use any obvious series and parallel connections to reduce the number of equations and variables.

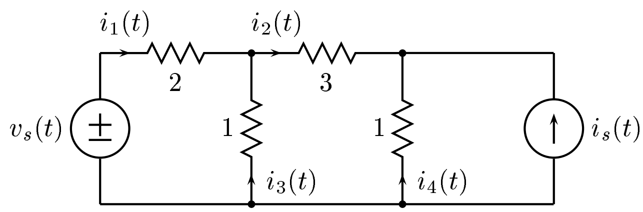


Figure 3: Circuit for Problem 1.3.

Solution: The right mesh KVL equation tells us only that the voltage drop across the current source is the same as the voltage drop across the resistor connected in parallel to it. The remaining two KVL equations are:

$$\begin{aligned} \text{left mesh:} \quad & 2i_1(t) - i_3(t) = v_s(t) \\ \text{center mesh:} \quad & i_3(t) + i_2(t) - i_4(t) = 0 \end{aligned}$$

Problem 1.4: Determine $v(t)$ and $i(t)$ in the network shown in Figure 4

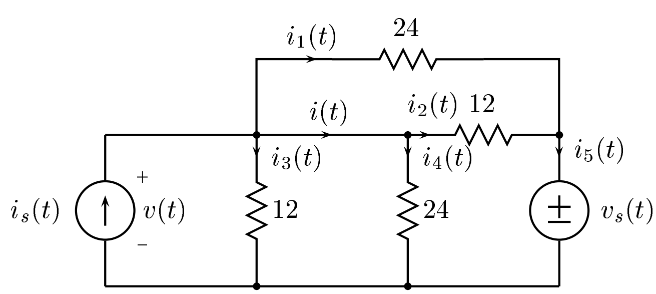


Figure 4: Circuit for Problem 1.4.

Solution: The two variables of interest, $v(t)$ and $i(t)$, can readily be expressed in terms of the element variables as

$$\begin{aligned} v(t) &= v_3(t) \\ i(t) &= i_2(t) + i_4(t). \end{aligned}$$

Therefore, it will be sufficient to solve for the resistor voltages and currents. If there are eight variables, we must have eight independent equations. We can get four of these from the element relations for the resistors

$$\begin{aligned} v_1(t) &= 24 i_1(t) \\ v_2(t) &= 12 i_2(t) \\ v_3(t) &= 12 i_3(t) \\ v_4(t) &= 24 i_4(t), \end{aligned}$$

one from KCL at the large node at the upper left

$$i_1(t) + i_2(t) + i_3(t) + i_4(t) = i_s(t),$$

and three from KVL equations (ignoring the mesh with the current source)

$$\begin{aligned} v_1(t) - v_2(t) &= 0 \\ v_3(t) - v_4(t) &= 0 \\ -v_2(t) + v_4(t) &= v_s(t). \end{aligned}$$

The four element relations will allow us to express the resistor voltages in terms of their currents. Furthermore, the first two KVL equations allow us to eliminate two of the currents since

$$\begin{aligned} v_1(t) = v_2(t) &\implies 2i_1(t) = i_2(t) \\ v_3(t) = v_4(t) &\implies 2i_4(t) = i_3(t). \end{aligned}$$

Thus the KCL equation reduces to

$$i_1(t) + i_4(t) = \frac{1}{3} i_s(t)$$

and the remaining KVL equation becomes

$$-i_1(t) + i_4(t) = \frac{1}{24} v_s(t).$$

These final equations are straightforward to solve. Adding the two equations gives

$$i_4(t) = \frac{1}{6} i_s(t) + \frac{1}{48} v_s(t).$$

Substituting this result into the first of these equations gives

$$i_1(t) = \frac{1}{6} i_s(t) - \frac{1}{48} v_s(t).$$

Since $2i_4(t) = i_3(t)$, we have

$$\boxed{v(t) = v_3(t) = 24 i_4(t) = 4 i_s(t) + \tfrac{1}{2} v_s(t)}$$

and

$$\boxed{i(t) = i_2(t) + i_4(t) = 2i_1(t) + i_4(t) = \tfrac{1}{2} i_s(t) - \tfrac{1}{48} v_s(t).}$$

Problem 1.5: The instantaneous power dissipated by an element or source is equal to the product of the voltage drop across the element, $v(t)$ and the current passing through it, $i(t)$

$$P_{inst}(t) = v(t)i(t).$$

It is important for this definition that the reference direction for the current flows into the + reference terminal for the voltage drop. The instantaneous power associated with an element can be either positive or negative; when it is positive, the element is dissipating power and when it is negative it is supplying power. Resistors always dissipate power.

Consider the circuit shown in Figure 5.

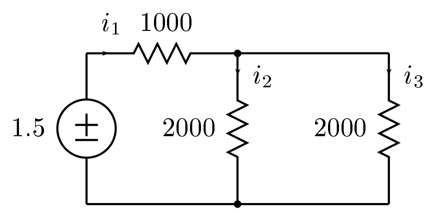


Figure 5: Circuit for Problem 1.5.

(a) This circuit contains three elements and thus there are six element variables, all of which are constant since the voltage source is constant. The currents are labeled and the voltage drops across the three resistors are implied by the default sign convention. Write a set of six linear equations in the variables v_1, v_2, v_3, i_1, i_2, and i_3 that specify the equilibrium solution. These should take the form of three element relations, one KCL equation, and two KVL equations.

(b) Solve the above set of equations to determine the equilibrium values of the element variables.

(c) Evaluate the power dissipated by all of the elements and sources. Show that the total power dissipated in the resistors is equal to the total power supplied by the source.

The net power in any circuit must always be zero, i.e. the total power dissipated must always equal the total power supplied. This problem demonstrates that fact for one particular circuit. We will prove the general case later in the course.

Solution:

(a)

$$\begin{aligned}
R_1 :&\quad v_1 = 1000 i_1 \\
R_2 :&\quad v_2 = 2000 i_2 \\
R_3 :&\quad v_3 = 2000 i_3 \\
KCL :&\quad i_1 - i_2 - i_3 = 0 \\
KVL_1 :&\quad v_1 + v_2 = 1.5 \\
KVL_2 :&\quad -v_2 + v_3 = 0
\end{aligned}$$

(b) The solution of these six equations in six unknowns is straightforward. The solution is:

$$v_1 = 0.75V \qquad i_1 = 0.75mA$$
$$v_2 = 0.75V \qquad i_2 = 0.375mA$$
$$v_3 = 0.75V \qquad i_3 = 0.375mA$$

(c)

$$P_1 = (0.75V)(0.75mA) = 0.5625mW$$
$$P_2 = (0.75V)(0.375mA) = 0.28125mW$$
$$P_3 = (0.75V)(0.375mA) = 0.28125mW$$

Therefore, the total power dissipated is:

$$P_d = P_1 + P_2 + P_3 = 1.125mW.$$

The total power supplied by the battery is

$$P_s = (1.5V)(0.75mA) = 1.125mW.$$
