GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

September 8, 2000

Problem Set #2–Solutions

Problem 2.1: Find the current $i_1(t)$ for the circuit shown in Figure 1.

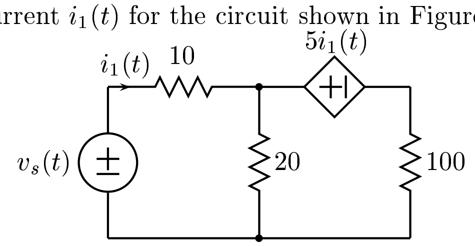


Figure 1: Circuit for Problem 2.1.

Solution: Define the auxiliary currents shown in Figure 2. From KCL

$$i_1(t) - i_2(t) - i_3(t) = 0 \tag{1}$$

From KVL applied to the left mesh

$$-v_s(t) + 10i_1(t) + 20i_2(t) = 0 \tag{2}$$

and from KVL applied to the right mesh

$$-20i_2(t) + 5i_1(t) + 100i_3(t) = 0. \tag{3}$$

Remember the dependent source is a *voltage* source with a voltage drop that is equal to $5i_1(t)$. From the first equation $i_3(t) = i_1(t) - i_2(t)$. Substituting into the second and third equations, this gives

$$10i_1(t) + 20i_2(t) = v_s(t)$$
$$105i_1(t) - 120i_2(t) = 0.$$

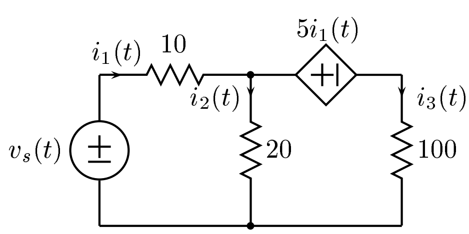


Figure 2: Circuit for Problem 1.26 with additional currents labeled.

We can now multiply the first equation by 6 and add the two equations together.

$$165 i_1(t) = 6 v_s(t)$$

or

$$i_1(t) = \frac{6}{165} v_s(t) = \frac{1}{55} v_s(t).$$

Problem 2.2: Determine the voltage $v(t)$ in the circuit in Figure 3.

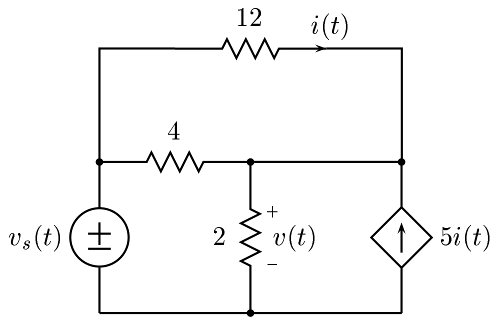
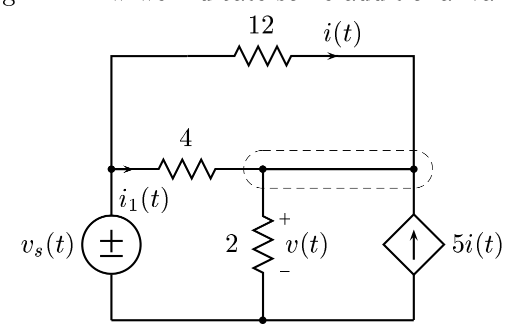


Figure 3: Figure for Problem 2.2.

Solution: In the figure below we indicate some additional variables.



Notice that we have let the current flowing through the 4Ω resistor be denoted by $i_1(t)$. The current flowing downward through the 2Ω resistor is $v(t)/2$. We can set up 1 KCL equation (at the circled node), and two KVL equations to solve for the three variables $i(t)$, $v(t)$, and $i_1(t)$.

$$\begin{aligned} \text{KCL:} &\quad i_1(t) + 6i(t) - \frac{1}{2}v(t) = 0 \\ \text{KVL } \alpha: &\quad 4i_1(t) + v(t) = v_s(t) \\ \text{KVL } \beta: &\quad 12i(t) - 4i_1(t) = 0. \end{aligned}$$

From the third equation

$$i_1(t) = 3i(t).$$

Substituting this fact into the first equation gives

$$9i(t) - \frac{1}{2}v(t) = 0 \Longrightarrow i(t) = \frac{1}{18}v(t).$$

Finally, substituting this result into the second equation gives

$$\frac{12}{18}v(t) + v(t) = v_s(t)$$

or

$$v(t) = \frac{3}{5}v_s(t).$$

Problem 2.3: This problem is concerned with the three networks shown in Figure 4.

(a) For the network in (a)

(i.) Draw the basic network.

(ii.) Identify the closed paths in the original network that correspond to meshes in the basic network.

(iii.) Identify the closed surfaces in the original network that correspond to nodes in the basic network.

(b) Repeat for the network in (b).

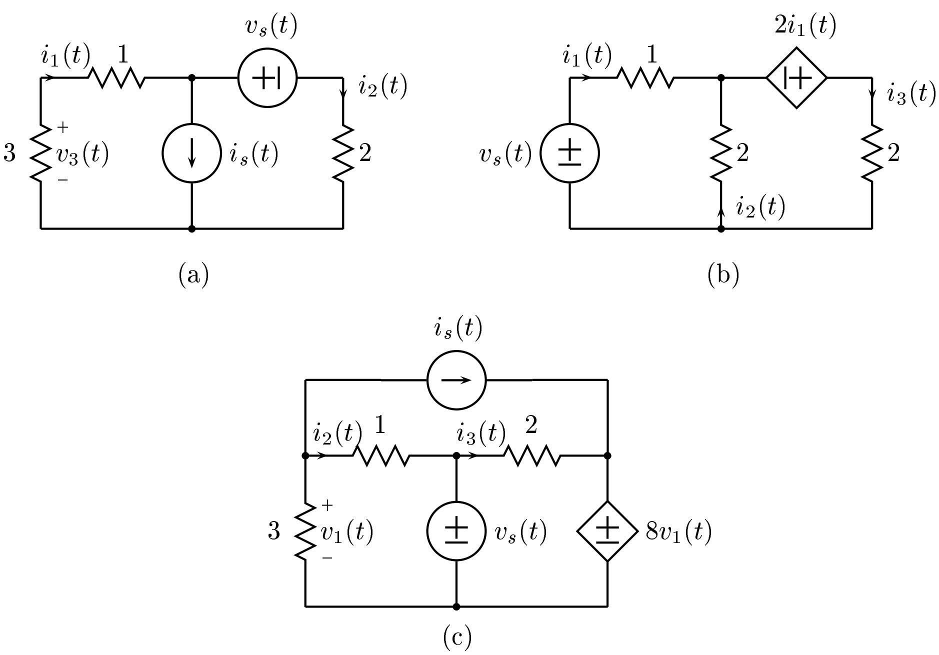


Figure 4: Networks for Problem 2.3.

(c) Repeat for the network in (c).

Solution:

(a) (i) The basic network is shown in Figure 5.

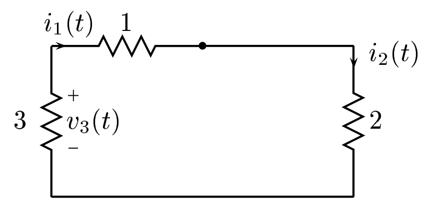


Figure 5: The basic network for the circuit in Figure 4(a).

(ii) There is only one mesh in the basic network and it corresponds to the path around the outside of the original network, i.e. it is the path that contains both the 3Ω and the 2Ω resistors.

(iii) The surfaces in the complete network that correspond to nodes in the basic network are indicated in Figure 6 as dashed lines.

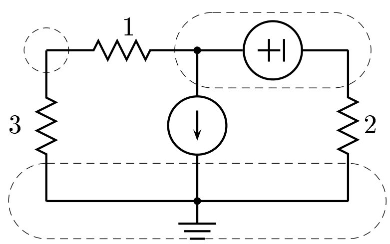


Figure 6: The surfaces in the complete network in Figure 4(a) corresponding to nodes in the basic network.

(b) (i) The basic network is shown in Figure 7.

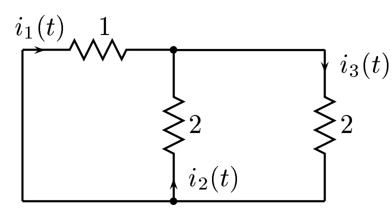


Figure 7: The basic network for the circuit in Figure 4(b).

(ii) The basic network contains two meshes. The paths corresponding to those meshes in the complete network are the two meshes in the complete network.

(iii) The basic network contains two nodes. In the complete network, those correspond to the two closed surfaces illustrated in Figure 8.

(c) (i) The basic network is shown in Figure 9.

(ii) The meshes in the basic network correspond to the two lower meshes in the complete network.

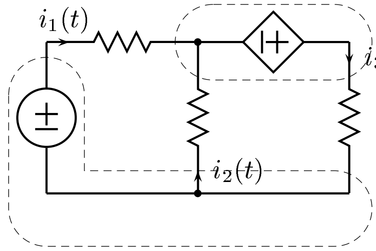


Figure 8: The surfaces in the complete network of Figure 4(b) corresponding to nodes in the basic network of Figure 7.

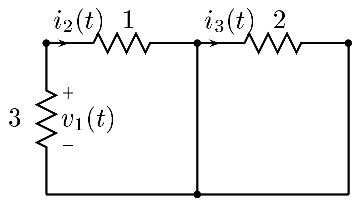


Figure 9: The basic network for the circuit in Figure 4(c).

(iii) The basic network contains only two nodes. In the complete network these correspond to the dashed surfaces shown in Figure 10. Since both terminals of the 2Ω resistor are included in that node, the resistor itself can be incorporated in it as well.

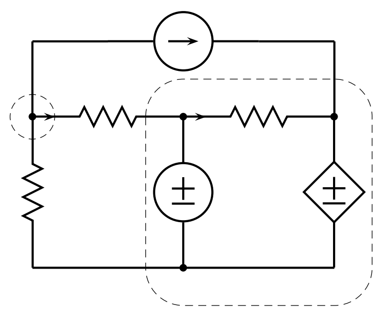


Figure 10: The surfaces for the network in Figure 4(c) corresponding to nodes in the basic network of Figure 7.

Problem 2.4: For each of the networks in Problem 2.3 above, write the complete set of linear equations that defines the equilibrium using the basic networks that you found. Put these equations in matrix-vector form and solve them for the indicated variables using MATLAB.

Solution: For each part of the problem, the circuit drawing is given in Problem

2.3 along with the nodes (surfaces) for writing the KCL equations and the paths for the KVL equations. In each case the lowest node on the drawing was the node omitted when writing the KCL equations. The equations were written in terms of the variables indicated on the circuit drawings.

(a) The equations that define the equilibrium solution are:

$$\begin{aligned} \text{KVL:} \quad & -v_3(t) + i_1(t) + 2i_2(t) = -v_s(t) \\ \text{KCL1:} \quad & -i_1(t) + \frac{1}{3}v_3(t) = 0 \\ \text{KCL2:} \quad & i_1(t) - i_2(t) = i_s(t). \end{aligned}$$

Putting these into matrix-vector form gives the matrix equation

$$\begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & \frac{1}{3} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} v_s(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} i_s(t).$$

MATLAB gives for the solution

$$\begin{aligned} i_1(t) &= \frac{1}{3}v_s(t) - \frac{2}{3}i_s(t) \\ i_2(t) &= \frac{1}{3}v_s(t) - \frac{5}{3}i_s(t) \\ v_3(t) &= v_s(t) - 2i_s(t). \end{aligned}$$

(b) The equations that define the equilibrium solution are:

$$\begin{aligned} \text{KVL1:} \quad & i_1(t) - 2i_2(t) = v_s(t) \\ \text{KVL2:} \quad & 2i_2(t) - 2i_1(t) + 2i_3(t) = 0 \\ \text{KCL (supernode):} \quad & i_1(t) + i_2(t) - i_3(t) = 0. \end{aligned}$$

Putting these into matrix-vector form gives

$$\begin{bmatrix} -1 & -2 & 0 \\ -2 & 2 & 2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_s(t).$$

MATLAB gives for the solution

$$\begin{aligned} i_1(t) &= v_s(t) \\ i_2(t) &= 0 \\ i_3(t) &= v_s(t). \end{aligned}$$

(c) The equations that define the equilibrium solution are:

$$\begin{aligned} \text{KCL:} \quad & -\frac{v_1(t)}{3} - i_2(t) = i_s(t) \\ \text{KCL1:} \quad & -v_1(t) + i_2(t) = -v_s(t) \\ \text{KCL2:} \quad & 2i_3(t) + 8v_1(t) = v_s(t). \end{aligned}$$

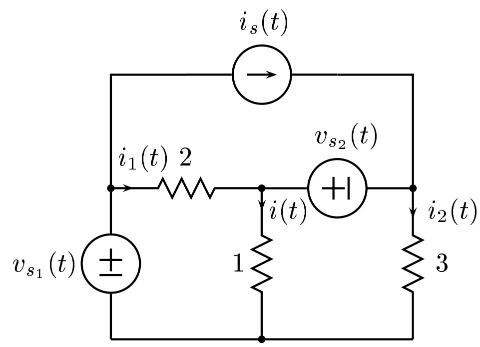
In matrix-vector form, these become

$$\begin{bmatrix} -\frac{1}{3} & -1 & 0 \\ -1 & 1 & 0 \\ 8 & 0 & 2 \end{bmatrix} \begin{bmatrix} v_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} v_s(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} i_s(t).$$

Notice that the first two equations allow us to find $v_1(t)$ and $i_2(t)$. The third equation simply gives the value for $i_3(t)$. MATLAB gives for the solution

$$\begin{aligned} v_1(t) &= \frac{3}{4}v_s(t) - \frac{3}{4}i_s(t) \\ i_2(t) &= -\frac{1}{4}v_s(t) - \frac{3}{4}i_s(t) \\ i_3(t) &= -\frac{5}{2}v_s(t) + 3i_s(t). \end{aligned}$$

Problem 2.5: Solve for $i(t)$ in the circuit below.



Solution: Define currents $i_1(t)$ and $i_2(t)$ as shown in the drawing above and define a supernode encircling the horizontal voltage source.

The basic network contains only two nodes and two meshes. We will, therefore, write two KVL equations (on the two lower meshes in the complete network) and one KCL equation on the supernode. We write all of these equations using the current variables.

$$\begin{aligned} \text{KCL:} &\quad i(t) - i_1(t) + i_2(t) = i_s(t) \\ \text{KVL1:} &\quad 2i_1(t) + i(t) = v_{s_1}(t) \\ \text{KVL2:} &\quad -i(t) + 3i_2(t) = -v_{s_2}(t). \end{aligned}$$

If we put these into matrix-vector form, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} i(t) \\ i_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} i_s(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v_{s_1}(t) + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} v_{s_2}(t).$$

The solution is

$$i(t) = 0.5455i_s(t) + 0.2727v_{s_1}(t) + 0.1818v_{s_2}(t)$$
