GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

September 15, 2000

Problem Set #3 – Solutions

Problem 3.1: Use the node method to write a set of equilibrium equations for the network of Figure 1. Use as variables the voltages $e_a(t)$, $e_b(t)$, and $e_c(t)$. Do not solve the set of equations.

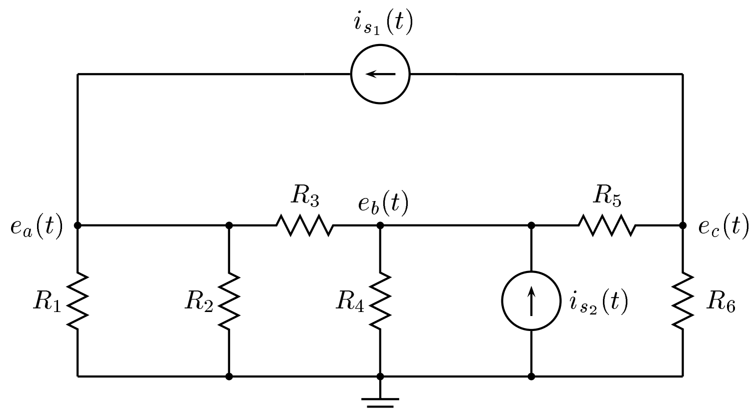

Figure 1: Circuit for Problem 3.1.

Solution:

$$\text{node } a: \quad \frac{1}{R_1}e_a(t) + \frac{1}{R_2}e_a(t) + \frac{1}{R_3}[e_a(t) - e_b(t)] = i_{s_1}(t)$$

$$\text{node } b: \quad \frac{1}{R_3}[e_b(t) - e_a(t)] + \frac{1}{R_4}e_b(t) + \frac{1}{R_5}[e_b(t) - e_c(t)] = i_{s_2}(t)$$

$$\text{node } c: \quad \frac{1}{R_5}[e_c(t) - e_b(t)] + \frac{1}{R_6}e_c(t) = -i_{s_1}(t)$$

Regrouping terms allows us to write these in the alternative form

$$\text{node } a: \quad \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] e_a(t) - \frac{1}{R_3}e_b(t) = i_{s_1}(t)$$

$$\text{node } b: \quad -\frac{1}{R_3}e_a(t) + \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] e_b(t) - \frac{1}{R_5}e_c(t) = i_{s_2}(t)$$

$$\text{node } c: \quad -\frac{1}{R_5}e_b(t) + \left[\frac{1}{R_5} + \frac{1}{R_6}\right] e_c(t) = -i_{s_1}(t)$$

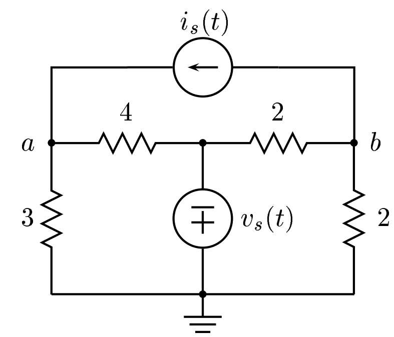

Figure 2: Circuit for Problem 3.2.

Problem 3.2: In this problem we shall solve the circuit in Figure 2 using the node method.

(a) Write the KCL equations at nodes a and b in terms of the node potentials at those nodes, $e_a(t)$ and $e_b(t)$.

(b) Put your equations in matrix-vector form by supplying the missing constants in the framework below.

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} i_s(t) + \begin{bmatrix} \quad \\ \quad \end{bmatrix} v_s(t)$$

(c) Solve them for $e_a(t)$ and $e_b(t)$.

Solution:

(a) Observe that the node potential of the node in the center of the circuit is $-v_s(t)$. With a potential defined at each node of the circuit the equations are particularly simple to write.

$$\frac{1}{3}e_a(t) + \frac{1}{4}[e_a(t) + v_s(t)] = i_s(t)$$
$$\frac{1}{2}e_b(t) + \frac{1}{2}[e_b(t) + v_s(t)] = -i_s(t)$$

(b)

$$\begin{bmatrix} \frac{7}{12} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} i_s(t) + \begin{bmatrix} -\frac{1}{4} \\ -\frac{1}{2} \end{bmatrix} v_s(t)$$

(c) These equations are particularly simply to solve

$$e_a(t) = \frac{12}{7} i_s(t) - \frac{3}{7} v_s(t)$$
$$e_b(t) = -i_s(t) - \frac{1}{2} v_s(t)$$

Problem 3.3: Determine the voltage at each connection point in the circuit in Figure 3 with respect to the indicated ground.

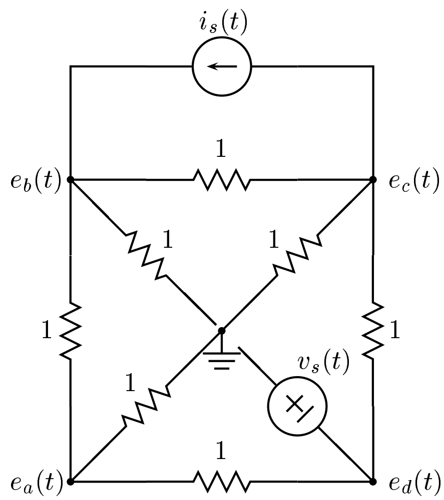

Figure 3: Circuit for Problem 3.3

Solution: We need to write a KCL equation at each of the three nodes of the basic network, a, b, and c. The node potential $e_d(t)$ is equal to $-v_s(t)$.

$$\begin{aligned}
\text{node } a: &\quad [e_a(t) - e_b(t)] + e_a(t) + [e_a(t) + v_s(t)] = 0 \\
\text{node } b: &\quad [e_b(t) - e_a(t)] + e_b(t) + [e_b(t) - e_c(t)] = i_s(t) \\
\text{node } c: &\quad [e_c(t) - e_b(t)] + e_c(t) + [e_c(t) + v_s(t)] = -i_s(t)
\end{aligned}$$

We can combine the three equations into one matrix-vector equation

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} e_s(t) \\ e_b(t) \\ e_c(t) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} v_s(t) + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} i_s(t).$$

The solution is

$$\begin{aligned}
e_a(t) &= -0.4286\, v_s(t) + 0.0952\, i_s(t) \\
e_b(t) &= -0.2857\, v_s(t) - 0.2857\, i_s(t) \\
e_c(t) &= -0.4286\, v_s(t) - 0.2381\, i_s(t) \\
e_d(t) &= -v_s(t).
\end{aligned}$$

Problem 3.4: We wish to solve the circuit in Figure 4 using the node method. Let $e_a(t)$ be the node potential at the indicated node.

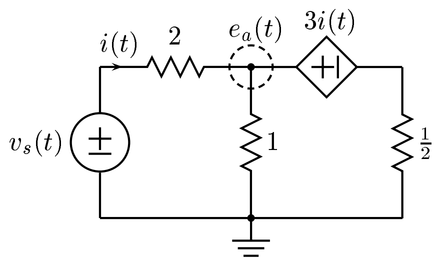

Figure 4: Circuit for Problem 3.4.

(a) Express $i(t)$ in terms of $e_a(t)$ and $v_s(t)$.

(b) Write a KCL equation at the surface in the complete network that corresponds to the non-ground node in the basic network. This equation should involve only the variables $e_a(t)$ and $v_s(t)$.

(c) Determine $e_a(t)$.

Solution:

(a) $i(t) = \frac{1}{2}\left[v_s(t) - e_a(t)\right]$

(b) This surface is a supernode that envelops the dependent voltage source. The potential at the negative terminal of the dependent source is $e_a(t) - 3i(t) = e_a(t) - \frac{3}{2}(v_s(t) - e_a(t)) = \frac{5}{2}e_a(t) - \frac{3}{2}v_s(t)$.

$$\frac{1}{2}[e_a(t) - v_s(t)] + e_a(t) + 2[\frac{5}{2}e_a(t) - \frac{3}{2}v_s(t)] = 0$$

Simplifying

$$\frac{13}{2}e_a(t) = \frac{7}{2}v_s(t).$$

(c) $e_a(t) = \frac{7}{13}v_s(t)$
