GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

September 29, 2000

Problem Set #5–Solutions

Problem 5.1: For the circuit in Figure 1 the source voltage on the left is a battery (i.e., a constant voltage source). Determine the value of V required to produce a value for V_o of 1 volt. For the

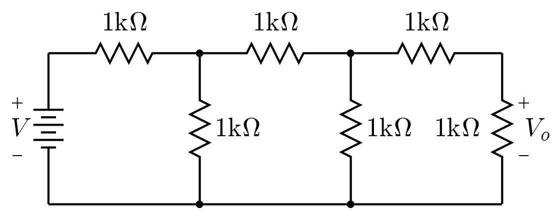


Figure 1: Circuit for Problem 5.1.

circuit in Figure 1 the source voltage on the left is a battery (i.e., a constant voltage source). Determine the value of V required to produce a value for V_o of 1 volt.

Solution: Number the six resistors R_1, R_2, etc. from left to right. To get 1 volt across R_6 requires a current flowing through it of 1 mA. Since this current also flows through R_5, there will be a 2 volt drop across the series combination of R_5 and R_6. This means that there will be a 2 volt drop across R_4 and a 2 mA current flowing through it. Working backwards, this means that there will be a 3 mA current through R_3 and a voltage drop of 3 volts. A 3 volt drop across R_3 with a 2 volt drop across R_4 means a 5 volt drop across R_2 and a current of 5 mA. Finally there will be an 8 mA current through R_1 and an 8 volt drop. Therefore, $V = 13$ volts. (Notice that the sequence of voltages [and the sequence of currents]forms a Fibonacci series.)

Problem 5.2:

(a) Find the Thévenin equivalent network corresponding to the two-terminal network in Figure 2.

(b) Find the Norton equivalent network.

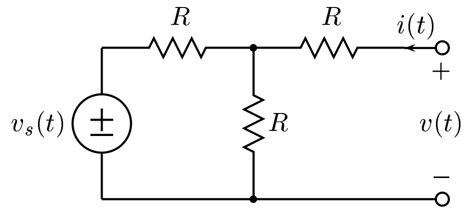
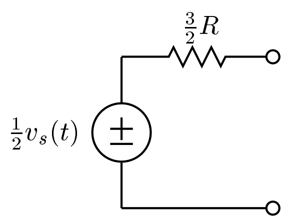


Figure 2: Circuit for Problem 5.2.

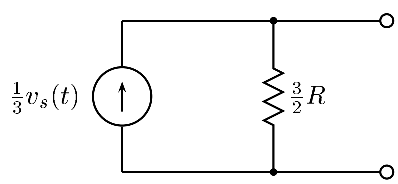
Solution:

$$v_{oc}(t) = \frac{1}{2} v_s(t)$$
$$R_{eq} = \frac{R}{2} + R = \frac{3}{2} R$$

(a)



(b)



Problem 5.3: Sketch the Thevenin equivalent circuit corresponding to the one-port in Figure 3.

Solution: Consider the 3Ω resistor at the top of the network. The potential at its right node is $10v_a(t)$ and the potential at its left node is $v_a(t)$. This means that there is a voltage drop (from right to left) across this resistor of $9v_a(t)$, and a current of $3v_a(t)$ also flowing from right to left. The current flowing down through the vertical 3Ω resistor is $v_a(t)/3$. Applying KCL at the output of the current source gives

$$i_s(t) + 3v_a(t) = \frac{1}{3} v_a(t)$$

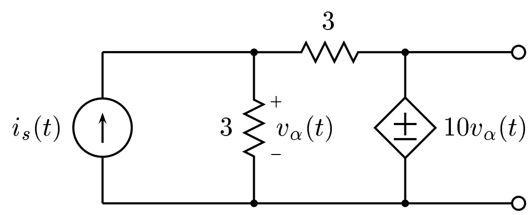


Figure 3: Circuit for Problem 5.3.

from which we see that

$$i_s(t) = -\frac{8}{3}v_a(t)$$

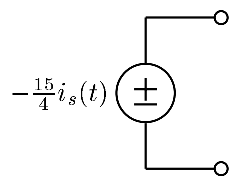
or

$$v_a(t) = -\frac{3}{8}i_s(t).$$

Therefore,

$$10v_a(t) = -\frac{15}{4}i_s(t).$$

The voltage at the terminals is a constant independent of the current flowing in. The Thévenin equivalent circuit is shown below



Problem 5.4: Consider a one-port network consisting of two capacitors with capacitances C_1 and C_2 connected in parallel, as shown in Figure 4.

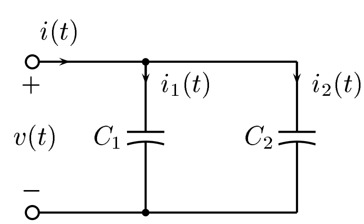


Figure 4: Two capacitors connected in parallel.

(a) Show that this network is equivalent to a single capacitor.

(b) Derive a formula for the equivalent capacitance C_{eq} in terms of C_1 and C_2.

(c) Derive expressions for the current $i_1(t)$ that passes through capacitor C_1 and the current $i_2(t)$ that passes through C_2 in terms of the current $i(t)$ entering the one-port.

Solution:

(a) By KCL we know

$$i(t) = i_1(t) + i_2(t),$$

but

$$i_1(t) = C_1 \frac{dv(t)}{dt}$$
$$i_2(t) = C_2 \frac{dv(t)}{dt}.$$

Therefore,

$$i(t) = C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} = (C_1 + C_2) \frac{dv(t)}{dt}.$$

(b) From the result of part (a)

$$C_{eq} = C_1 + C_2.$$

(c) From (a)

$$i_1(t) = C_1 \frac{dv(t)}{dt} = C_1 \left(\frac{i(t)}{C_1 + C_2} \right) = \frac{C_1}{C_1 + C_2} i(t).$$
$$i_2(t) = C_2 \frac{dv(t)}{dt} = C_2 \left(\frac{i(t)}{C_1 + C_2} \right) = \frac{C_2}{C_1 + C_2} i(t).$$

Problem 5.5:

Determine $v_{out}(t)$ in terms of $v_{in}(t)$ for the circuit in Figure 5.

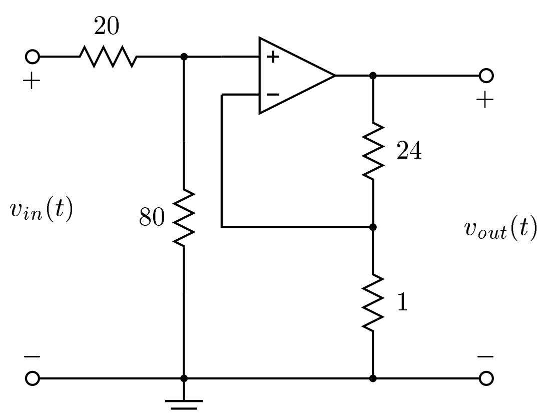


Figure 5: Circuit for Problem 5.5.

Solution: Circuits containing opamps are normally analyzed using the node method, appropriately modified. For this circuit we need to write KCL equations at the two nodes connected to the two inputs of the opamp. The potential at each of these is the same; call that potential $e(t)$.

$$\begin{aligned} +\text{ node:} \quad & \frac{1}{20}[e(t) - v_{in}(t)] + \frac{1}{80}e(t) = 0 \\ -\text{ node:} \quad & \frac{1}{24}[e(t) - v_{out}(t)] + e(t) = 0 \end{aligned}$$

From the first equation we find

$$e(t) = \frac{4}{5}v_{in}(t).$$

Substituting this value into the second equation gives

$$\begin{aligned} & \frac{1}{24}[\frac{4}{5}v_{in}(t) - v_{out}(t)] + \frac{4}{5}v_{in}(t) = 0 \\ & \frac{5}{6}v_{in}(t) - \frac{1}{24}v_{out}(t) = 0 \\ & v_{out}(t) = 20v_{in}(t). \end{aligned}$$

Problem 5.6: Find the voltage gain of the circuit in Figure 6. The voltage gain is defined as $G = v_{out}(t)/v_{in}(t)$.

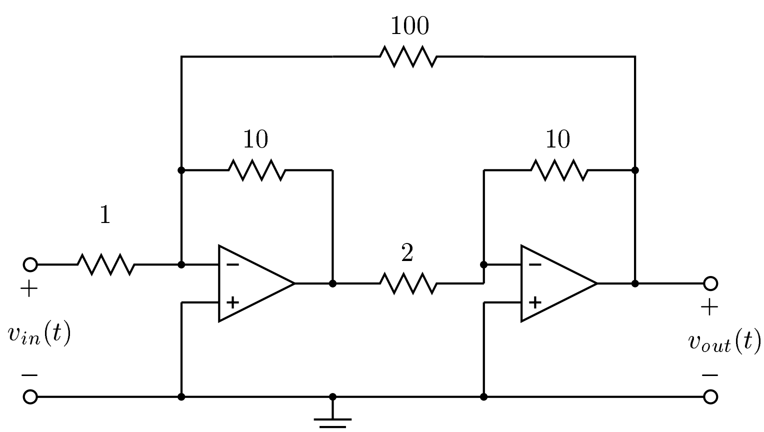


Figure 6: Circuit for Problem 5.6.

Solution: We need two KCL equations at the two nodes attached to the inverting inputs of the two operational amplifiers. The potential at each of these nodes is zero because it is a virtual ground. Let the potential at the output node of the left operational amplifier by $e(t)$; the potential at the output of the second is $v_{out}(t)$. At the input to the left opamp we have

$$v_{in}(t) + \frac{1}{10}e(t) + \frac{1}{100}v_{out}(t) = 0.$$

At the input to the right opamp we have

$$\frac{1}{2}e(t) + \frac{1}{10}v_{out}(t) = 0.$$

From the second equation

$$e(t) = -\frac{1}{5}v_{out}(t).$$

If we substitute this result into the first equation we find

$$v_{out}(t) = 100v_{in}(t),$$

from which we get G=100.