GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

October 6, 2000

Problem Set #6–Solutions

Problem 6.1:

(a) Design a circuit with four inputs $v_1(t)$, $v_2(t)$, $v_3(t)$ and $v_4(t)$ containing a single operational amplifier, such that the output voltage $v_{out}(t)$ satisfies

$$v_{out}(t) = k_5[k_1 v_1(t) + k_2 v_2(t) - k_3 v_3(t) - k_4 v_4(t)]$$

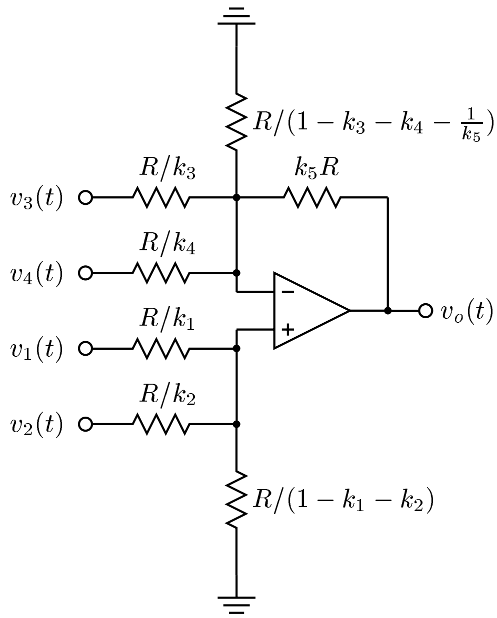
for positive values of the four constants. *Hint:* You might begin by designing a circuit that combines features of the differential amplifier configuration and the summing amplifier configuration.

(b) Prove that your design in (a) works correctly.

(c) State any additional constraints that need to be imposed on the five constants for the circuit to be buildable.

Solution: The approach suggested by the hint for this problem is to combine the structures for the differential amplifier and the summing amplifier and then through trial and error to work out a set of values for the resistors that will give the desired input-output relation. It is easiest if all of the resistors are proportional to some base resistance R.

(a) One circuit that will work is the following



(b) Let the potential at both the inverting and non-inverting inputs of the opamp be $e(t)$. Then from KCL at the non-inverting node:

$$\frac{e(t)-v_1(t)}{R/k_1}+\frac{e(t)-v_2(t)}{R/k_2}+\frac{e(t)}{R/(1-k_1-k_2)}=0.$$

This equation reduces to

$$e(t)=k_1v_1(t)+k_2v_2(t). \tag{1}$$

From KCL at the inverting node

$$\frac{e(t)-v_3(t)}{R/k_3}+\frac{e(t)-v_4(t)}{R/k_4}+\frac{e(t)-v_0(t)}{k_5R}+\frac{e(t)-v_1(t)}{R/(1-k_3-k_4-\frac{1}{k_5})}=0$$

This one reduces to

$$e(t)=\frac{1}{k_5}v_o(t)+k_4v_4(t)+k_3v_3(t) \tag{2}$$

Equating these two equations for $e(t)$ [(1) and (2)]gives

$$\frac{1}{k_5}v_o(t)+k_4v_4(t)+k_3v_3(t)=k_1v_1(t)+k_2v_2(t)$$

or

$$v_o(t)=k_5[k_1v_1(t)+k_2v_2(t)-k_3v_3(t)-k_4v_4(t)]$$

(c) All of the resistor values must be positive. Thus, we must have

$$\begin{aligned}&k_5>1\\&k_1>0;\ k_2>0;\ k_3>0;\ k_4>0\\&k_1+k_2<1\\&k_3+k_4+\frac{1}{k_5}<1\end{aligned}$$

Problem 6.2: Express the output voltage $v_{out}(t)$ in terms of the input voltage $v_{in}(t)$ for the circuit in Figure 1.

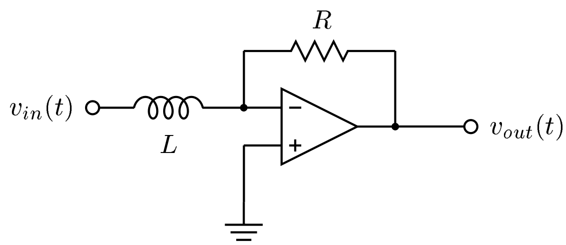


Figure 1: Circuit for Problem 6.2.

Solution: Let the current entering the node connected to the inverting input through the inductor be $i_\ell(t)$. Then the KCL equation at that node is

$$i_\ell(t) + \frac{1}{R} v_{out}(t) = 0$$

Substituting the $v - i$ relation for the inductor (in integral form gives

$$\frac{1}{L}\int_{t_0}^{t} v_{in}(\beta)\, d\beta + i_\ell(t_0) + \frac{1}{R} v_{out}(t) = 0,$$

which we can solve for $v_{out}(t)$.

$$v_{out}(t) = -\frac{R}{L}\int_{t_0}^{t} v_{in}(\beta)\, d\beta - R i_\ell(t_0).$$

Problem 6.3: Determine the Laplace transforms of the following time waveforms.

(a) $x_a(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$

(b) $x_b(t) = t^2 e^{-3t}, \;\; t > 0$

(c) $x_c(t) = e^{-4t} \sin 5t, \;\; t > 0$

(d) $x_d(t) = t, \;\; t > 0$

Solution:
(a)

$$X_a(s) = \int_0^T 1 \cdot e^{-st}\, dt = \frac{1}{s}\left(1 - e^{-sT}\right)$$

(b) To get $X_b(s)$ we can use an indirect approach (or we could just do the integral).

$$\begin{aligned}
z(t) = e^{-3t} \quad &\longleftrightarrow \quad \frac{1}{s+3} \\
y(t) = te^{-3t} \quad &\longleftrightarrow \quad -\frac{d}{ds}\left(\frac{1}{s+3}\right) = \frac{1}{(s+3)^2} \\
x_b(t) = ty(t) \quad &\longleftrightarrow \quad -\frac{d}{ds}\left(\frac{1}{(s+3)^2}\right) = \frac{2}{(s+3)^3}
\end{aligned}$$

(c)

$$\begin{aligned}
x_c(t) &= e^{-4t}\sin 5t = \frac{1}{j2}\left(e^{-4t}e^{j5t} - e^{-4t}e^{-j5t}\right) \\
&= \frac{1}{j2}e^{-(4-j5)t} - \frac{1}{j2}e^{-(4+j5)t} \\
X_c(s) &= \frac{\frac{1}{j2}}{s+4-j5} - \frac{\frac{1}{j2}}{s+4+j5} \\
&= \frac{5}{s^2+8s+41}
\end{aligned}$$

(d)

$$\begin{aligned}
x_d(t) &= t = 1 \cdot t \\
X_d(s) &= -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}
\end{aligned}$$

Problem 6.4: Find the inverse Laplace transform of

$$X(s) = \frac{2s+6}{s(s^2+3s+2)}.$$

Solution:

$$\begin{aligned}
X(s) &= \frac{2s+6}{s(s^2+3s+2)} = \frac{2s+6}{s(s+1)(s+2)} \\
&= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}
\end{aligned}$$

We can evaluate A, B, and C using

$$A = \lim_{s \to 0} \frac{2s+6}{(s+1)(s+2)} = 3$$
$$B = \lim_{s \to -1} \frac{2s+6}{s(s+2)} = -4$$
$$C = \lim_{s \to -2} \frac{2s+6}{s(s+1)} = 1$$

Therefore,

$$x(t) = 3 - 4e^{-t} + e^{-2t}, \quad t > 0.$$
