GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

October 20, 2000

Problem Set #8–Solutions

Problem 8.1: Determine $v_{out}(t)$ for $t > 0$ for the circuit in Figure 1 if $v_{out}(0) = 0$.

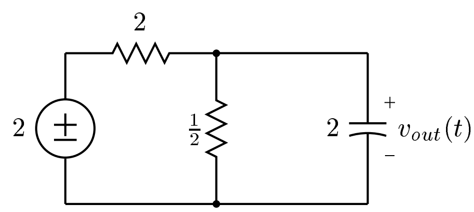
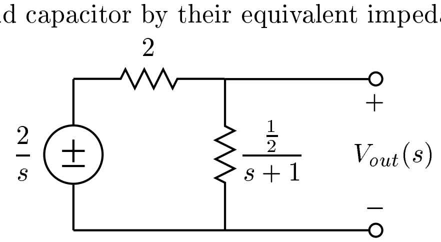


Figure 1: Circuit for Problem 8.1.

Solution: We begin by redrawing the circuit in the Laplace domain and replacing the parallel resistor and capacitor by their equivalent impedance.



We can now determine $V_{out}(s)$ by using a voltage divider.

$$V_{out}(s) = \frac{\frac{\frac{1}{2}}{s+1}}{2 + \frac{\frac{1}{2}}{s+1}} \cdot \frac{2}{s} = \frac{\frac{1}{2}}{s(s+\frac{5}{4})}$$
$$= \frac{A}{s} + \frac{B}{s+\frac{5}{4}}$$

$$A = \lim_{s \to 0} \frac{\frac{1}{2}}{s+\frac{5}{4}} = \frac{2}{5}$$
$$B = \lim_{s \to -\frac{5}{4}} \frac{\frac{1}{2}}{s} = -\frac{2}{5}$$

Therefore,

$$v_{out}(t) = \frac{2}{5}(1 - e^{-1.25t}), \quad t > 0$$

Problem 8.2: (a) For the circuit in Figure 2, determine $v_c(t)$ if $i_s(t) = 1$ for $t > 0$ under the assumption that $v_c(0) = 0$.

(b) Repeat for $v_c(0) = 1$.

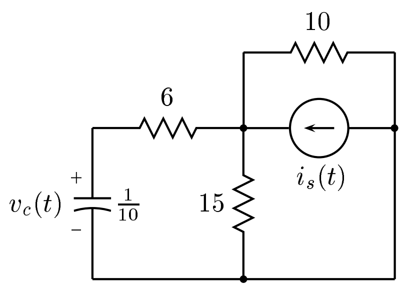
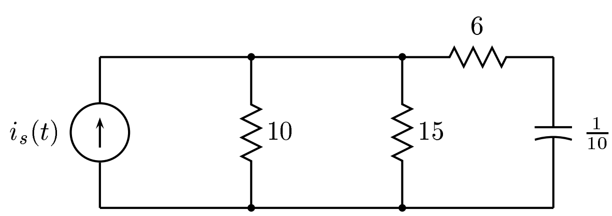
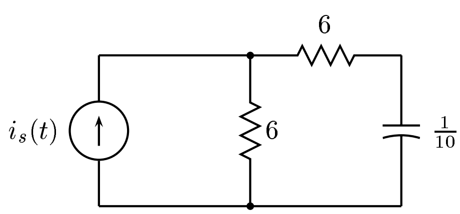


Figure 2: Circuit for Problem 8.2.

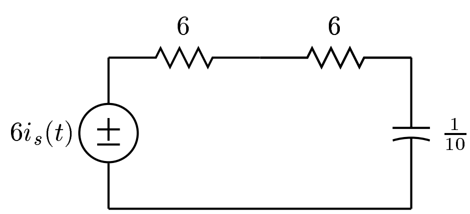
Solution: As an alternative to setting up and solving a set of equations to determine $V_c(s)$, we can simplify the circuit by replacing the current source and resistors by their Thevenin equivalent. First, we redraw the circuit to give it a more familiar appearance



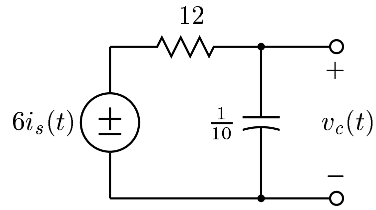
After combining the parallel resistors this becomes



Now we can replace the current source in parallel with a resistor with a resistor in series with a voltage source. This gives



Finally, this becomes



Converting to the Laplace domain and applying a voltage divider

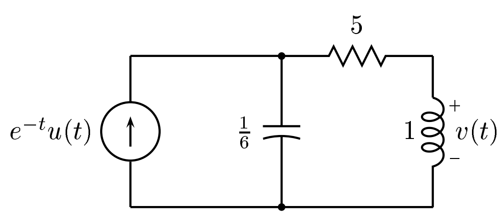
$$V_c(s) = \frac{\frac{10}{s}}{12 + \frac{10}{s}} \cdot \frac{6}{s} = \frac{5}{s(s + \frac{5}{6})}$$
$$= \frac{A}{s} + \frac{B}{s + \frac{5}{6}}$$

$$A = \lim_{s \to 0} \frac{5}{s + \frac{5}{6}} = 6$$
$$B = \lim_{s \to -\frac{5}{6}} \frac{5}{s} = -6$$

Therefore,

$$v_c(t) = 6(1 - e^{-5t/6}), \quad t > 0.$$

Problem 8.3: In the circuit below solve for $v(t)$ for $t > 0$ if $v_c(0) = 0$ and $i_\ell(0) = 0$, where $v_c(t)$ is the voltage drop across the capacitor and $i_\ell(t)$ is the current through the inductor.



Solution: Clearly

$$V(s) = sI_\ell(s)$$

and we can determine $I_\ell(s)$ by using a current divider. Therefore,

$$V(s) = s\left(\frac{\frac{6}{s}}{s + 5 + \frac{6}{s}}\right)\left(\frac{1}{s+1}\right) = \frac{6s}{(s+1)(s+2)(s+3)}.$$

Performing a partial fraction expansion gives

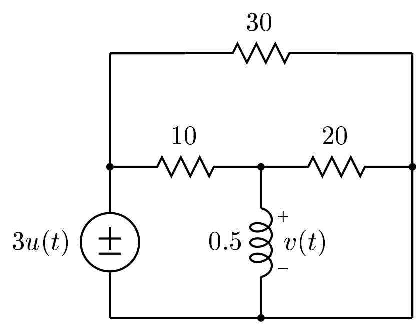
$$V(s) = \frac{-3}{s+1} + \frac{12}{s+2} + \frac{-9}{s+3}$$

and

$$v(t) = \left(-3e^{-t} + 12e^{-2t} - 9e^{-3t}\right)u(t).$$

The response is zero for $t < 0$ because the input is zero in that range.

Problem 8.4: Consider the first-order circuit below



(a) Find $v(0)$.

(b) Find $v(\infty)$.

(c) Find $v(t)$ for $t > 0$.

Solution:

(a) At $t = 0$ the inductor looks like an open circuit. The voltage drop across the inductor is the same as the voltage drop across the 20 Ohm resistor. Using a voltage divider, this is seen to be 2 Volts.

$$v(0) = 2V$$

(b) At $t = \infty$, the inductor becomes a short circuit. The voltage drop across it is then zero.

$$v(\infty) = 0.$$

(c) To get the Thevenin resistance, we turn off the voltage source, replacing it by a short circuit. The inductor is then seen to be connected in parallel with the 10 Ohm resistor and also with the 20 Ohm resistor. (The 30 Ohm resistor is shorted out.) Therefore

$$R_T = \frac{20}{3}$$
$$\tau = \frac{L}{R_T} = \frac{3}{40}$$

and

$$v(t) = 2e^{-40t/3} \quad t > 0.$$

Problem 8.5: The circuit in Figure 3 is at initial rest, i.e. the capacitor voltage is zero at $t = 0$, and $v_s(t) = 1$ for $t > 0$.

(a) Determine $v_{out}(0)$.

(b) Determine $v_{out}(\infty)$.

(c) Determine $v_{out}(t)$ for all t.

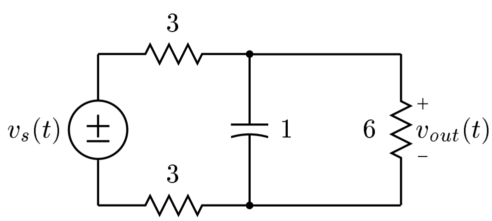


Figure 3: Circuit for Problem 8.5.

Solution:

(a) At $t = 0$ the capacitor looks like a short circuit. Therefore,

$$v_{out}(0) = 0.$$

(b) At $t = \infty$ the capacitor looks like an open circuit. Therefore, by a voltge divider

$$v_{out}(\infty) = \frac{1}{2}.$$

(c) The time constant $\tau = R_{eq}C = 3$, since when the voltage source is turned off, the capacitor is connected in parallel with two 6Ω resistors. Therefore,

$$v_{out}(t) = \frac{1}{2}\left(1 - e^{-t/3}\right) u(t).$$
