GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

October 27, 2000

Problem Set #9–Solutions

Problem 9.1: For each of the networks in Figure 1, determine the equivalent impedance. Express your answers as ratios of polynomials in s.

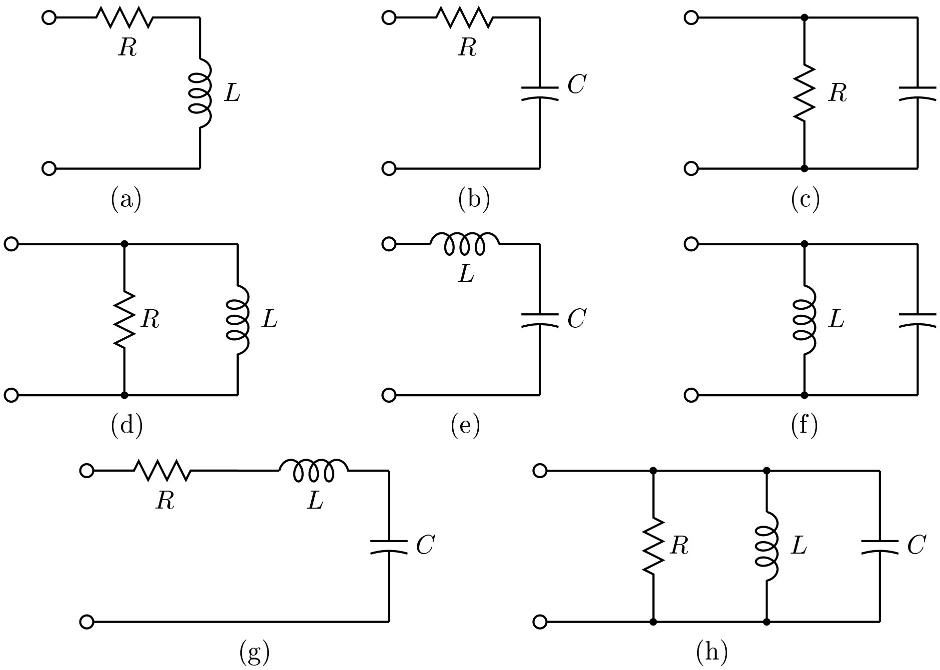


Figure 1: Circuits for Problem 9.1.

Solution:

(a) $Z_a(s) = R + Ls$

(b) $Z_b(s) = R + \frac{1}{CS} = \frac{RCs+1}{Cs}$

(c) $Z_c(s) = \frac{\frac{R}{Cs}}{R+\frac{1}{Cs}} = \frac{R}{RCs+1}$

(d) $Z_d(s) = \frac{RLs}{R+Ls}$

(e) $Z_e(s) = Ls + \frac{1}{Cs} = \frac{LCs^2+1}{Cs}$

(f) $Z_f(s) = \frac{\frac{L}{C}}{Ls+\frac{1}{Cs}} = \frac{LCs^2+1}{Cs}$

(g) $Z_g(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs}$

(h) $Z_h(s) = \frac{1}{\frac{1}{R}+\frac{1}{Ls}+Cs} = \frac{RLs}{RLCs^2+Ls+R}$

Problem 9.2: Find the Laplace domain Thevenin equivalent network that corresponds to the one-port circuit in Figure 2 at initial rest.

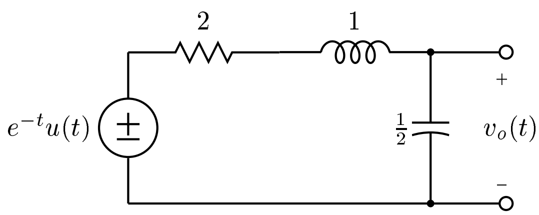


Figure 2: Circuit for Problem 9.2.

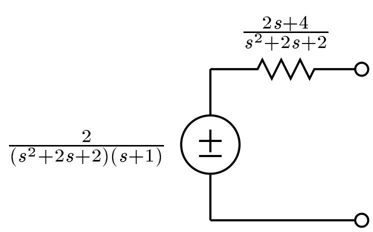
Solution: To find the Thevenin equivalent network in the Laplace domain, we simply need to find the equivalent impedance with the voltage source turned off and the open-circuit voltage.

$$Z_T(s) = \frac{\frac{2}{s}\cdot(s+2)}{2+2+\frac{2}{s}} = \frac{2s+4}{s^2+2s+2}$$

To get the open circuit voltage, we can apply a voltage divider

$$\begin{aligned} V_{oc}(s) &= \frac{\frac{2}{s}}{s+2+\frac{2}{s}}\cdot\frac{1}{s+1} \\ &= \frac{2}{(s^2+2s+2)(s+1)} \end{aligned}$$

This gives the equivalent network (Laplace domain)



Problem 9.3: Find $v_{out}(t)$ for $t > 0$ when $v_{in}(t) = \cos(1000t)$ and $v_{out}(0) = 0$ for the circuit drawn in Figure 3.

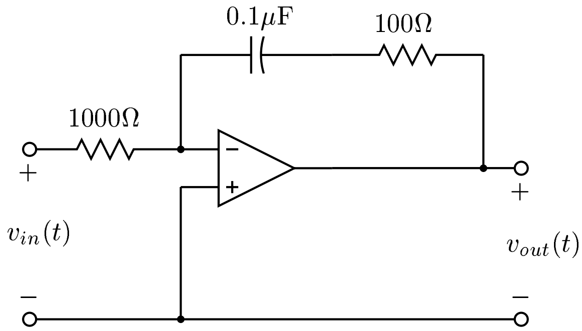


Figure 3: Circuit for Problem 9.3.

Solution: This circuit looks like an inverting amplifier with

$$Z_i(s) = 1000$$
$$Z_f(s) = \frac{1}{10^{-5}s} + 100 = \frac{10^5}{s} + 100.$$

Therefore,

$$V_{out}(s) = -\frac{Z_f(s)}{Z_i(s)} \cdot v_{in}(s)$$
$$= -\frac{\frac{10^5}{s} + 100}{1000} V_{in}(s) = \left(\frac{100}{s} + 0.1\right) v_{in}(s).$$

Since

$$V_{in}(s) = \frac{s}{s^2 + 10^6}$$

we have

$$V_{out}(s) = \frac{100}{s^2 + 10^6} + \frac{0.1s}{s^2 + 10^6}.$$

From the table of Laplace transforms and the linearity property

$$v_{out}(t) = 0.1 \sin 1000t + 0.1 \cos 1000t, \quad t > 0.$$

Problem 9.4: When $i_s(t) = 2e^{-2t}$ in the circuit in Figure 4, the voltage drop across the inductor is observed to be $v_\ell(t) = 2e^{-4t} - e^{-2t}$ for $t > 0$. Determine the values of R and L.

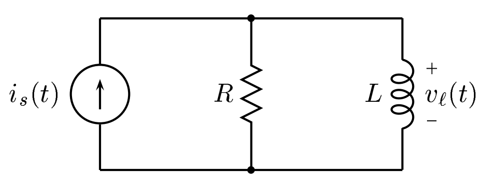


Figure 4: A circuit with unknown values of R and L for Problem 9.4.

Solution: One approach that will work here is to solve the circuit with generic values of R and L and then compare the result with the given result.

$$\begin{aligned} V_\ell(s) &= \frac{RLs}{R + Ls} \cdot I_s(s) \\ &= \frac{RLs}{R + Ls} \cdot \frac{2}{s+2}. \end{aligned}$$

Before proceding with the partial fraction expansion, we observe that $R/L = 4$. Substituting this fact now will simplify the algebra.

$$\begin{aligned} V_\ell(s) &= \frac{RLs}{R + Ls} \cdot \frac{2}{s+2} \\ &= \frac{4R}{s+4} + \frac{-2R}{s+2}. \end{aligned}$$

Therefore,

$$v_\ell(t) = 4Re^{-4t} - 2Re^{-2t}, \quad t \geq 0.$$

Comparing this with the given observation, we see

$$R = \frac{1}{2}.$$

Therefore, since $R/L = 4$, we must have

$$L = \frac{1}{8}.$$

Problem 9.5: Find the system function $H(s) = V_{out}(s)/V_{in}(s)$ for the circuit in Figure 5.

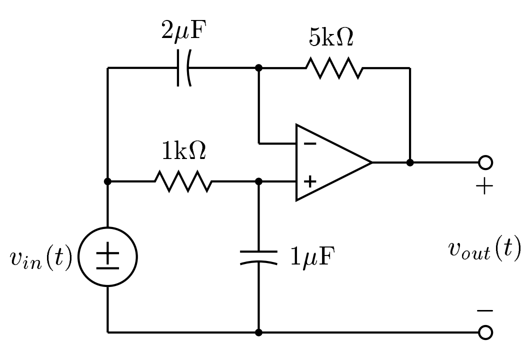


Figure 5: Circuit for Problem 9.5.

Solution: Let the node potentials at the two nodes connected to the inputs of the opamp be $E(s)$. (They are the same.) Then we get for KCL equations

$$C_1 s[E(s) - V_{in}(s)] + \frac{1}{R_2}[E(s) - V_{out}(s)] = 0$$
$$\frac{1}{R_1}[E(s) - V_{in}(s)] + C_2 s E(s) = 0$$

From the second equation

$$E(s) = \frac{1}{(1 + R_1 C_2 s)} V_{in}(s).$$

From the first

$$[1 + C_1 R_2 s] E(s) - R_2 C_1 s V_{in}(s) = V_{out}(s)$$
$$\left[\frac{1 + C_1 R_2 s}{1 + C_2 R_1 s} - R_2 C_1 s\right] V_{in}(s) = V_{out}(s)$$

From this

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1 - C_1 C_2 R_1 R_2 s^2}{1 + C_2 R_1 s} = \frac{1 - 10^{-5} s^2}{1 + 10^{-3} s}$$
