GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

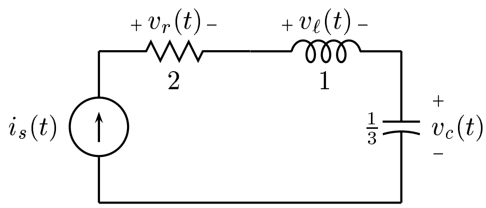
ECE 2040
Circuit Analysis

Quiz #1 – Solutions

Problem Q1.1: For the circuit below the current source waveform is

$$i_s(t) = \begin{cases} e^{-t} - e^{-2t}, & t > 0 \\ 0, & t < 0 \end{cases}$$

It is known that $v_c(-\infty) = 0$.



(a) What is $v_r(t)$?

(b) What is $v_\ell(t)$?

(c) What is $v_c(t)$?

Solution: The same current $i_s(t)$ flows through all three elements.

(a) $v_r(t) = Ri_r(t) = 2i_s(t)$. Therefore,

$$v_r(t) = \begin{cases} 2(e^{-t} - e^{-2t}), & t > 0 \\ 0, & t < 0. \end{cases}$$

(b) $v_\ell(t) = L\frac{di_s(t)}{dt}$. Therefore,

$$v_\ell(t) = \begin{cases} -e^{-t} + 2e^{-2t}, & t > 0 \\ 0, & t < 0. \end{cases}$$

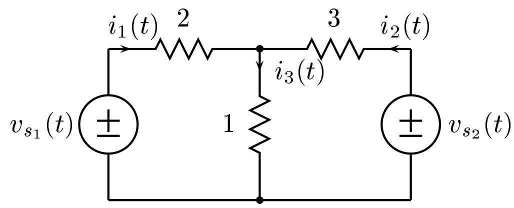
(c) Since

$$v_c(t) = \frac{1}{C}\int_0^t i_c(\tau)\, d\tau.$$

Since $v_c(-\infty) = 0$ and $i_c(t) = 0$, $t < 0$, we must have $v_c(0) = 0$. This specifies the constant of integration.

$$v_c(t) = \begin{cases} -3e^{-t} + \frac{3}{2}e^{-2t} + \frac{3}{2}, & t > 0 \\ 0, & t < 0 \end{cases}$$

Problem Q1.2:



(a) For the above circuit write a sufficient set of KCL equations needed to find the equilibrium solution. These should be in terms of the variables $i_1(t)$, $i_2(t)$, and $i_3(t)$.

(b) Write a sufficient set of KVL equations to find the equilibrium solution. These should also be in terms of the variables $i_1(t)$, $i_2(t)$, and $i_3(t)$.

(c) Express your complete set of equations in matrix-vector form. You do not need to solve them.

Solution: Since we recognize that $i_1(t)$ and $i_2(t)$ are the currents flowing through the two voltage sources, we need only write one KCL equation and two KVL equations.
(a)

$$i_1(t) + i_2(t) - i_3(t) = 0$$

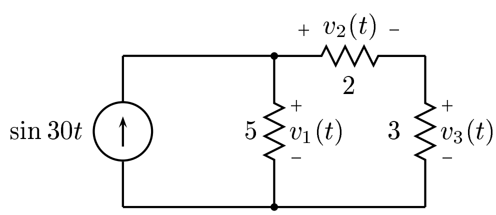
(b)

$$\begin{aligned} \text{KVL1:} \quad & 2i_1(t) + i_3(t) = v_{s_1}(t) \\ \text{KVL2:} \quad & -i_3(t) + 3i_2(t) = -v_{s_2}(t) \end{aligned}$$

(c) In matrix-vector form these become

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v_{s_1}(t) + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} v_{s_2}(t).$$

Problem Q1.3: Solve the circuit below for $v_1(t)$, $v_2(t)$, and $v_3(t)$.



Solution: We need to write 2 KCL equations and one KVL equation. If we write these in terms of the voltage variables, we save time.

$$\begin{aligned}\text{KCL1:}\quad & \frac{v_1(t)}{5} + \frac{v_2(t)}{2} = \sin 30t \\ \text{KCL2:}\quad & \frac{v_2(t)}{2} - \frac{v_3(t)}{3} = 0 \\ \text{KVL:}\quad & -v_1(t) + v_2(t) + v_3(t) = 0\end{aligned}$$

From the second equation we know that $v_3(t) = \frac{3}{2}v_2(t)$. Substituting this fact into the third equation gives

$$-v_1(t) + v_2(t) + \frac{3}{2}v_2(t) = 0$$

from which we learn that $v_1(t) = \frac{5}{2}v_2(t)$. Substituting this fact into the first equation gives

$$\frac{v_2(t)}{2} + \frac{v_2(t)}{2} = \sin 30t$$

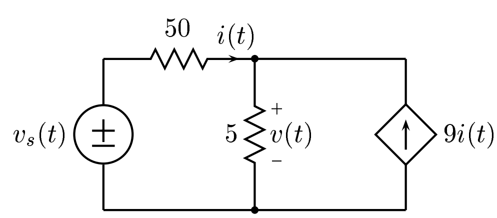
or

$$v_2(t) = \sin 30t.$$

Therefore,

$$\begin{aligned}v_1(t) &= \frac{5}{2}\sin 30t \\ v_3(t) &= \frac{3}{2}\sin 30t.\end{aligned}$$

Problem Q1.4:



In the circuit above the only variables of interest are $i(t)$ and $v(t)$.

(a) Using the basic network as your guide write a minimal set of KCL equations in terms of $i(t)$ and $v(t)$ to specify the equilibrium solution.

(b) Write a similar set of KVL equations.

(c) Solve these for $i(t)$ and $v(t)$.

Solution:

(a) Since the basic network contains only two nodes, we need only one KCL equation. We will get the same equation from either node:

$$i(t) - \frac{v(t)}{5} + 9i(t) = 0.$$

Rewriting allows us to replace this by

$$10i(t) - \frac{v(t)}{5} = 0.$$

(b) The basic network contains only a single mesh. Therefore, we need only one KVL equation.

$$50i(t) + v(t) = v_s(t).$$

(c) To solve these, multiply the KCL equation by 5 and add the two equations together. This gives

$$100i(t) = v_s(t)$$

or

$$i(t) = \frac{1}{100} v_s(t).$$

Substituting this result into either of the two original equations gives

$$v(t) = \frac{1}{2} v_s(t).$$
