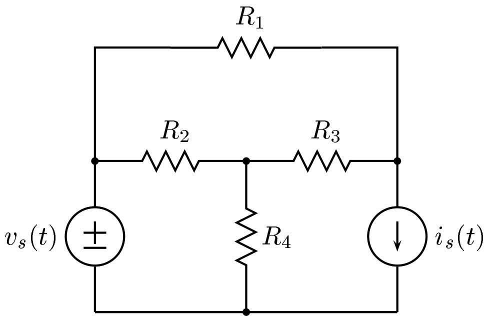
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #2–Solutions

Problem Q2.1:



(a) For the circuit above, what is the minimum number of KCL equations that need to be written to specify the equilibrium solution?

(b) Select one of the nodes of the basic network as the ground node and label it. Define node potentials at the remaining nodes of the basic network and write a KCL equation at each of these nodes in terms of the node potentials (and the source waveforms).

(c) Put your equations in matrix-vector form with the node potentials as unknowns. You do not need to solve the equations.

Solution:

(a) The basic network contains three nodes. Therefore, we need to write at least two KCL equations.

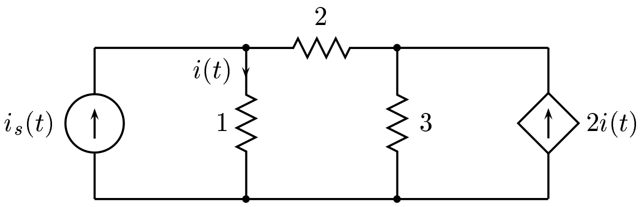
(b) Select the (super)node at the bottom as the ground. We then write KCL equations at the two nodes connected to resistor R_3. Let these node potentials be denoted $e_a(t)$ (left node) and $e_b(t)$ (right node). Then the two KCL equations are

$$\frac{1}{R_2}[e_a(t) - v_s(t)] + \frac{1}{R_4}e_a(t) + \frac{1}{R_3}[e_a(t) - e_b(t)] = 0$$
$$\frac{1}{R_3}[e_b(t) - e_a(t)] + \frac{1}{R_1}[e_b(t) - v_s(t)] = -i_s(t)$$

(c)

$$\begin{bmatrix} \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_1} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{R_2} \\ \frac{1}{R_1} \end{bmatrix} v_s(t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} i_s(t)$$

Problem Q2.2:



(a) For the circuit above, what is the minimum number of KVL equations that need to be written to specify the equilibrium solution?

(b) Define a mesh current for each mesh of the basic network. Write a sufficient set of KVL equations in terms of the mesh currents and the source waveforms.

(c) Solve your equations and use the results to determine $i(t)$.

Solution:

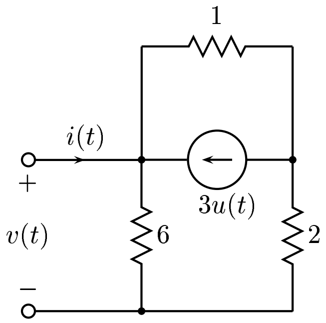
(a) There is one mesh in the basic network. Therefore, we need only a single KVL equation.

(b) Let $i_\alpha(t)$ be the mesh current in the center mesh.

$$[i_\alpha(t) - i_s(t)] + 2i_\alpha(t) + 3[i_\alpha(t) + 2(i_s(t) - i_\alpha(t))] = 0$$

(c) Unfortunately, because of a particularly poor choice of element values, the above equation is singular. It is satisfied only for the unreasonable value, $i_\alpha = \infty$, which produces an infinite value for $i(t)$. Grading for this part of the problem is based on the reasonableness of your approach.

Problem Q2.3:



(a) For the above two-terminal network, determine the open-circuit voltage $v_{oc}(t)$.

(b) Determine the short-circuit current, $i_{sc}(t)$.

(c) Determine and sketch the Norton equivalent network.

Solution:

(a) When the terminals are open circuited, the current source sees an 8Ω resistor connected in parallel with a 1Ω one. The current flowing through the 6Ω resistor is

$$\frac{1}{1+8}3u(t) = \frac{1}{3}u(t)$$

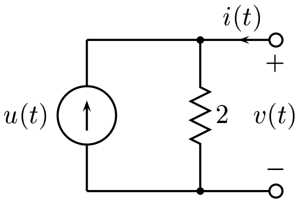
and the voltage drop across that resistor is

$$v_{oc}(t) = 6 \cdot \frac{1}{3}u(t) = 2u(t).$$

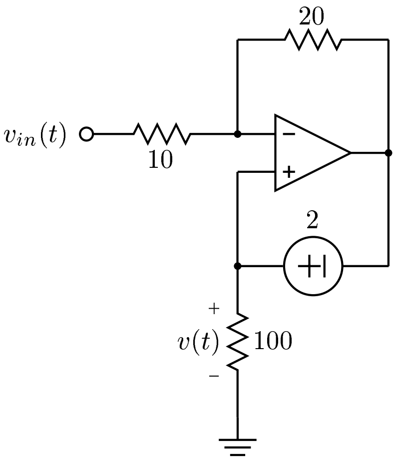
(b) When the terminals are short circuited, the current source sees a 2Ω resistor connected in parallel with a 1Ω one. The short-circuit current is the current flowing through the 2Ω resistor, which is one-third of the current coming from the source. (Be careful with the sign!)

$$i_{sc}(t) = -u(t)$$

(c) The Norton equivalent network is sketched below.



Problem Q2.4:



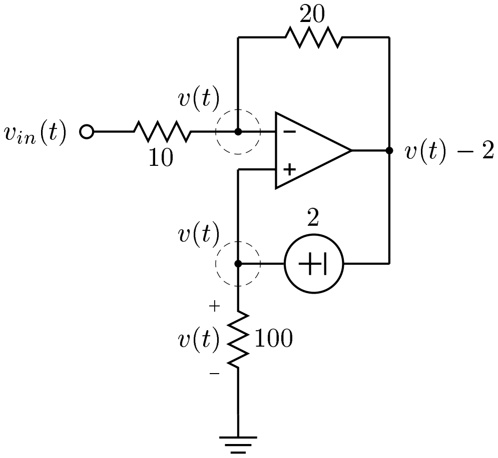
(a) Circle the nodes at which you can write valid KCL equations.

(b) Label the potential of each node in the circuit.

(c) Solve for $v(t)$.

Solution:

(a,b) The nodes are circled and the node potentials are labeled in the following figure.



(c) Writing a KCL equation at the upper of the two valid nodes gives:

$$\frac{v(t) - v_{in}(t)}{10} + \frac{v(t) - v(t) + 2}{20} = 0$$

or

$$v(t) = v_{in}(t) - 1.$$
