GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #3–Solutions

Problem Q3.1: The inverse Laplace transform of

$$X(s) = \frac{s+2}{(s+2)^2+9}$$

has the form

$$x(t) = Ke^{-at}\cos(\omega_0 t + \phi).$$

Determine the values of K, a, ω_0, and ϕ.

Solution: The partial fraction expansion has the form

$$X(s) = \frac{A}{s+2-j3} + \frac{A^*}{s+2+j3}.$$

To determine A we evaluate the limit

$$A = \lim_{s\to -2+j3} \frac{s+2}{s+2+j3} = \frac{j3}{j6} = \frac{1}{2}.$$

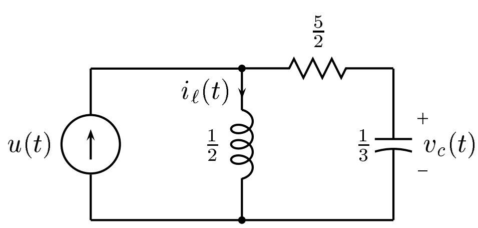
Therefore,

$$x(t) = \frac{1}{2}e^{-2t}e^{j3t} + \frac{1}{2}e^{-2t}e^{-jt} = e^{-2t}\cos 3t.$$

Thus,

$$K = 1, \quad a = 2, \quad \omega_0 = 3, \quad \phi = 0.$$

Problem Q3.2:



$i_\ell(0) = 0$
$v_c(0) = 2$

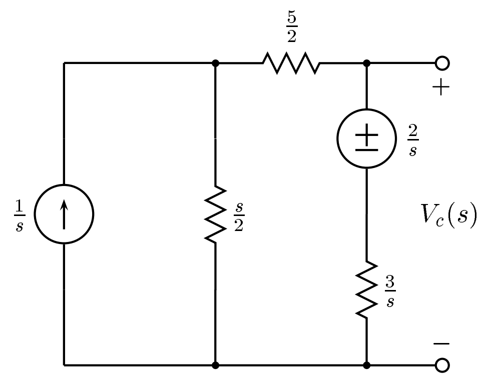
(a) Sketch the mapping of the circuit to the Laplace domain.

(b) Compute the Laplace transform of the capacitor voltage, $V_c(s)$.

(c) Compute $v_c(t)$ for $t \geq 0$.

Solution:

(a) The Laplace domain circuit is shown in the next figure.



(b) Using superposition of sources

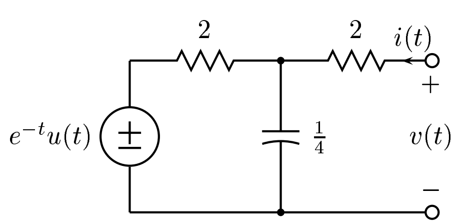
$$\begin{aligned} V_c(s) &= \frac{s}{3} \cdot \frac{\frac{s}{2}}{\frac{s}{2} + \frac{5}{2} + \frac{3}{s}} \cdot \frac{1}{s} + \frac{2}{s} \cdot \frac{\frac{5}{2} + \frac{s}{2}}{\frac{s}{2} + \frac{5}{2} + \frac{3}{s}} \\ &= \frac{s^2/3}{s^2 + 5s + 6} + \frac{s+5}{s^2+5s+6} \\ &= \frac{1}{3} + \frac{\frac{13}{3}}{s+2} - \frac{5}{s+3}. \end{aligned}$$

(c)

$$v_c(t) = \frac{1}{3}\delta(t) + \frac{13}{3}e^{-2t} - 5e^{-3t}, \quad t \geq 0.$$

Problem Q3.3:

The two-terminal network above is at initial rest.



(a) Compute the Laplace transform of the open-circuit voltage $V_{oc}(s)$ for the above circit.

(b) Compute the Laplace transform of the short-circuit current $I_{sc}(s)$ for the other circuit.

(c) Determine and sketch a Laplace-domain Thevenin equivalent network corresponding to the two-terminal network above.

Solution:
(a)

$$V_{oc}(s) = \frac{\frac{4}{s}}{2 + \frac{4}{2}} \cdot \frac{1}{s+1} = \frac{2}{s+2} \cdot \frac{1}{s+1}.$$

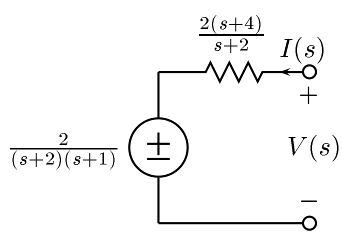
(b)

$$Z_T(s) = 2 + \frac{2 \cdot \frac{4}{s}}{2 + \frac{4}{s}} = 2 + \frac{4}{s+2} = \frac{2(s+4)}{s+2}.$$

Therefore,

$$I_{sc}(s) = -\frac{V_{oc}(s)}{Z_T(s)} = -\frac{2}{(s+2)(s+1)} \cdot \frac{s+2}{2(s+4)} = -\frac{1}{(s+1)(s+4)}.$$

(c) The circuit is sketched below



Problem Q3.4:

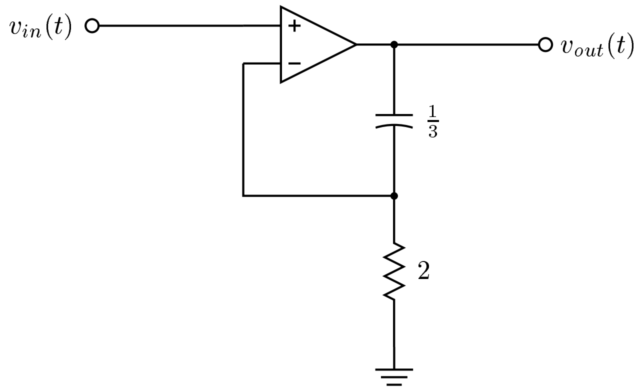
(a) The system function of the above circuit, which is at initial rest, has the form

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{as+b}{s+c}.$$

Determine the values of a, b, and c.

(b) Determine $v_{out}(t)$ for all t, if $v_{in}(t) = e^{-2t}u(t)$.

Solution:



(a) The potential at the node that connects the resistor to the capacitor is $V_{in}(s)$. Writing a KCL equation at that node gives

$$\frac{V_{in}(s)}{2} + \frac{s}{3}[V_{in}(s) - V_{out}(s)] = 0.$$

Simplifying, we get

$$\begin{aligned} V_{in}(s)\left[\frac{1}{2} + \frac{s}{3}\right] &= \frac{s}{3}V_{out}(s) \\ V_{out}(s) &= \frac{3}{s}\left[\frac{2s+3}{6}\right]V_{in}(s) \end{aligned}$$

and

$$H(s) = \frac{s + \frac{3}{2}}{s}.$$

Therefore, $a = 1$, $b = 3/2$, and $c = 0$.

(b)

$$V_{in}(s) = \frac{1}{s+2}$$

Therefore,

$$V_{out}(s) = \frac{s + \frac{3}{2}}{s(s+2)} = \frac{\frac{3}{4}}{s} + \frac{\frac{1}{4}}{s+2}.$$

$$v_{out}(t) = \left[\frac{3}{4} + \frac{1}{4}e^{-2t}\right]u(t)$$

Notice that the output is zero before $t = 0$.
