GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #4–Solutions

Problem Q4.1: A circuit with input source waveform $x_{in}(t)$ and output $y_{out}(t)$ has the system function

$$H(s) = \frac{s}{(s+2)^2 + 100}.$$

Determine $y_{out}(t)$, if

$$x_{in}(t) = 3\cos(10t).$$

Solution: Since $x_{in}(t) = \Re\mathrm{e}(3e^{j10t})$, we know that

$$\begin{aligned} y_{out}(t) &= \Re\mathrm{e}(3H(j10)e^{j10t}) = \Re\mathrm{e}\left(\frac{j30}{4+j40}e^{j10t}\right) \\ &= \Re\mathrm{e}\left(0.7463e^{\tan^{-1}(0.1)e^{j10t}}\right) \\ &= 0.7463\cos(10t + \tan^{-1}(0.1)). \end{aligned}$$

Problem Q4.2:

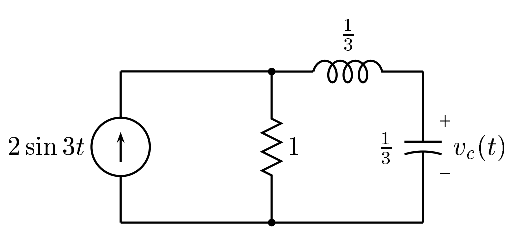


Figure 1: Circuit for Problem Q4.2.

(a) Find the system function of the circuit in Figure 1.

(b) Find $v_c(t)$ for all t.

Solution:

(a) The system function is

$$H(s) = \frac{V_c(s)}{I_s(s)} = \frac{3}{s}\frac{1}{\frac{1}{3}s + 1 + \frac{3}{s}} = \frac{9}{s^2 + 3s + 9}$$

(b) Since

$$H(j3) = \frac{1}{j} = -j = e^{-\frac{\pi}{2}},$$

we have

$$v_c(t) = \Im\mathrm{m}(2e^{j3t-\frac{\pi}{2}}) = -2\cos(3t).$$

Problem Q4.3: A circuit has the pole-zero plot shown in Figure 2.

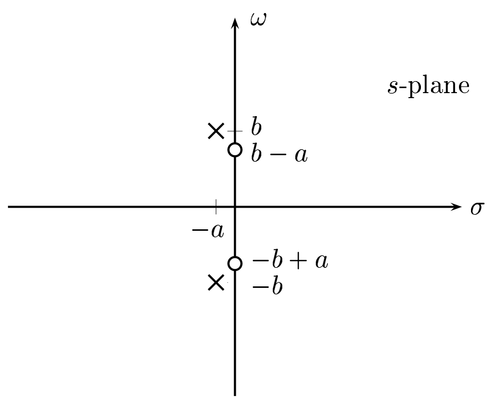


Figure 2: A pole-zero plot for Problem Q4.3.

(a) Graph the magnitude response of the filter. Indicate key frequencies on your plot.

(b) Determine the phase at the following frequencies:

(i) $\omega = 0$.

(ii) $\omega = \infty$.

(iii) $\omega = b - a - \epsilon$, ϵ = a very small number.

(iv) $\omega = b - a + \epsilon$.

(v) $\omega = a + b$.

Solution:

(a) The magnitude is plotted in Figure 3.

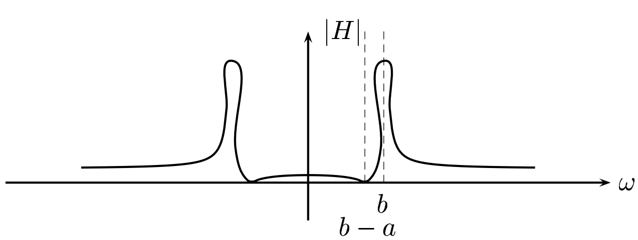


Figure 3: Magnitude response for Problem Q4.3.

(b) (i) $\angle H(j0) = \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} = 0$.

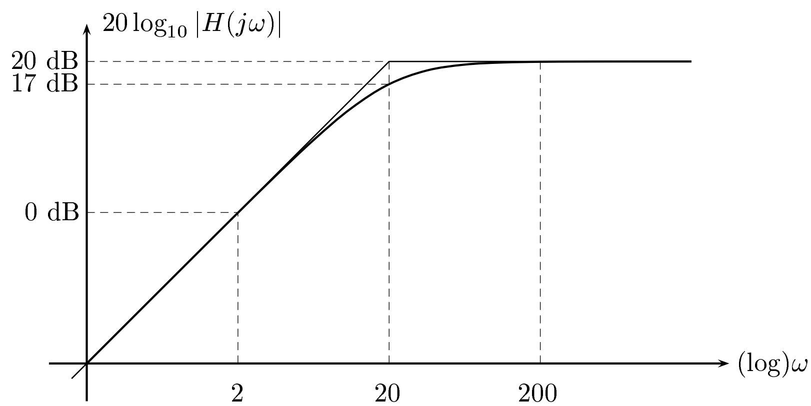
(ii) $\angle H(j\infty) = \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} = 0$.

(iii) At this frequency the phase should be slightly negative.

(iv) As the frequency response passes through the zero the phase should jump up by π radians. Thus the phase should be π greater than the result in (c).

(v) $\angle H(j(a+b)) \approx \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$.

Problem Q4.4: A stable circuit (i.e. one with all of its poles in the left half of the s-plane) has the Bode magnitude plot shown below.



(a) Determine the locations of all of the poles and zeros of the circuit.

(b) Determine the system function of the circuit, $H(s)$.

(c) Draw the Bode phase plot (asymptotes only). Clearly label your plot.

Solution:

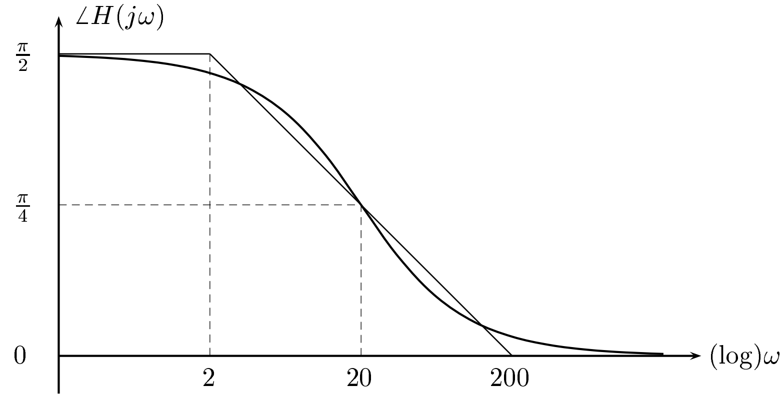
(a) There is a break frequency at 20 rad/s at which the slope is reduced from 20 dB/decade to 0 dB/decade. This implies a pole at $s = -20$. There are no other breakpoints. The slope of 20 dB/decade extending to the left is caused by a zero at the origin. Therefore, there is a single zero at $s = 0$ and a single pole at $s = -20$.

(b)

$$H(s) = \frac{10s}{s+20}$$

From the magnitude plot, the high frequency gain of the filter is 10 (=20 dB). Thus we need the constant 10 in the numerator.

(c) From the system function we can plot the phase response.



$\angle H(j\omega)$
$\frac{\pi}{2}$
$\frac{\pi}{4}$
0
2
20
200
(log)ω