GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 6258
Digital Image Processing
Fall 2003

Problem Set #3

Issued: Friday, September 12, 2003
Due (live): Monday, September 22, 2003
Due (video): Monday, October 6, 2003

Problem 3.1 (2-D Recursive Systems): A linear shift-invariant two-dimensional system is defined by the difference equation

$$y[n_1, n_2] - 0.7y[n_1 + 1, n_2] + 0.2y[n_1 + 1, n_2 + 1] = x[n_1, n_2]$$

(a) Determine the system function $H_z(z_1, z_2)$.

(b) If the filter is implemented using the recursion equation

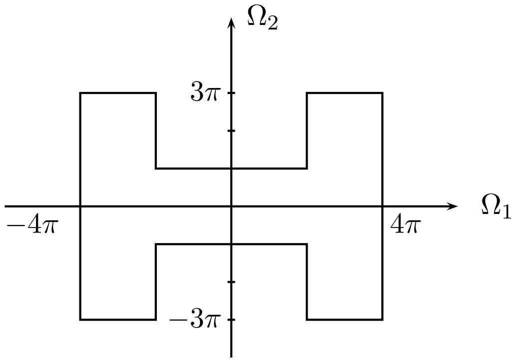
$$y[n_1, n_2] = x[n_1, n_2] + 0.7y[n_1 + 1, n_2] - 0.2y[n_1 + 1, n_2 + 1]$$

determine the region of support of the impulse response, $h[n_1, n_2]$, i.e. those values of $[n_1, n_2]$ where the impulse response can be nonzero.

(c) Specify a sufficient set of boundary values for evaluating $y[n_1, n_2]$ if $x[n_1, n_2]$ is nonzero for $0 \leq n_1 \leq N_1 - 1$, $0 \leq n_2 \leq N_2 - 1$.

(d) Determine whether or not the filter is stable.

Problem 3.2 (Image Sampling): In a feeble attempt to unify the nation's sampling grids, Hannibal Hamlin once decreed that henceforth all analog imagery in the country should have the spectral support (bandwidth) indicated below. Fortunately, the decree was struck down by the Supreme Court before it would have become effective.



(a) Determine an aliasing matrix $\boldsymbol{U}$ that will permit periodic replication of the spectrum without aliasing in a way that will minimize the required sampling density.

(b) Determine a consistent sampling matrix $\boldsymbol{V}$ that would define an optimal sampling lattice for this spectral support.

(c) What is the sampling density at which all of the country's images would have been sampled?

Problem 3.3 (Video Sampling): A (somewhat boring) analog video scene consists of a single object $o(x, y)$ moving in front of a black background with constant velocity. The resulting video signal, $s(x, y, t)$ can thus be modeled as

$$s(x, y, t) = o(x - v_x t, y - v_y t),$$

where v_x and v_y are the horizontal and vertical components of the velocity, measured in mm/s. The **two-dimensional** Fourier transform of $o(x, y)$ is bandlimited with a circularly-shaped region of support with a radius of W rad/mm.

(a) Determine the (continuous-variable) 3-D Fourier transform of $s(x, y, t)$ in terms of the quantities that have been defined.

(b) If the video signal can be represented exactly from its samples, determine a sufficient sampling lattice and describe a system that can be used to recover the video signal from its samples. If it cannot be represented exactly, explain why not. (*Note: This problem requires some thought.*)

Problem 3.4 (The 2-D Discrete Fourier Series (DFS)): Suppose that $\tilde{x}[n_1, n_2]$ is a rectangularly periodic sequence with horizontal period N_1 and vertical period N_2. The sequence $\tilde{x}_1[n] = \tilde{x}[n, n]$ is then a periodic **one-dimensional** sequence.

(a) Show that $\tilde{x}[n]$ is a periodic sequence with period $N_1 N_2$. Show that if N_1 and N_2 have any common integral factors then $\tilde{x}[n]$ will also have a smaller period. (*Hint:* Draw some pictures.)

(b) Assuming that N_1 and N_2 have no common factors, show that the samples of the DFS coefficients $\tilde{X}_1[k]$ are equal to selected values of $\tilde{X}[k_1, k_2]$ and determine the mapping between k and $[k_1, k_2]$.

If the row-column algorithm is used to evaluate the 2-D DFS coefficients, $\tilde{X}[k_1, k_2]$ this is an efficient algorithm for computing the one-dimensional DFS $\tilde{X}_1[k]$ known as the *prime factor algorithm.* It predates the Cooley-Tukey (FFT) algorithm and is more efficient than that algorithm, but places an inconvenient restriction on the length of the transform.