GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 6258
Digital Image Processing
Fall 2003

Problem Set #5

Issued: Monday, October 20, 2003
Due (live): Wednesday, October 29, 2003
Due (video): Wednesday, November 10, 2003

Problem 5.1 (Blockwise DCT) The DCT-II is a separable unitary transform, so that the DCT of an image block of size $M \times M$ is $X = A^T x A$, where x is the $M \times M$ image block and A is the $M \times M$ transform matrix containing elements

$$a_{ik} = \alpha_i \cos\left(\frac{(2k+1)i\pi}{2M}\right) \quad \text{for } i, k = 0, 1, \ldots, M-1$$

with

$$\alpha_0 = \sqrt{\frac{1}{M}}$$
$$\alpha_i = \sqrt{\frac{2}{M}} \quad \forall i > 0.$$

Implement a DCT of block size 8×8 and its inverse. Print out the values of the matrix A. Since we do not care about fast algorithms here, you can simply use matrix multiplications in MATLAB.

Problem 5.2 (Uniform Quantizer) Implement quantization of the coefficients by a uniform (mid-tread) quantizer with for an arbitrary step size Q. Use the same step-size for all coefficients. Using the BOATS image on the class website ('boats.gif'), measure the mean-squared quantization error for $Q = 2^0, 2^1, 2^2, 2^3, 2^4, ..., 2^7$.

Problem 5.3 (Entropy of the DCT coefficients) To estimate the bit-rate required to encode the coefficients, we will assume that each coefficient is coded at a bit-rate equal to its entropy, but that each of the 64 DCT coefficients has a different entropy (fractions are OK). Thus we need to determine the entropy for coefficient (0,0), the entropy for coefficient (1,0), etc. These entropies will vary with the amount of quantization that is applied to the DCTs. The average number of bits/pixel is then equal to 1/64*sum(entropy of each coefficient). Determine the bit-rate for $Q = 2^0, 2^1, 2^2, 2^3, 2^4, ..., 2^9$, using block DCTs from the images BOATS, MANDRILL, and PEPPERS, which are available on the web site. (It is OK to use the MATLAB function `dct2` instead of your function in Problem 5.1 to speed up the program.)

Problem 5.4 (Rate-distortion Function) The PSNR will be used to measure the quality of the reconstructed images. It is defined for 8-bit images as follows:

$$PSNR = 10 \log_{10}\left(\frac{255^2}{d}\right) \quad \text{[dB]}$$

The average distortion, d is the mean-squared error between the original and reconstructed images. Using the same quantizers and the same images that you used in the previous problem, produce a graph showing distortion as a function of bit-rate. This graph should show the PSNR we should expect as a function of bit-rate. (*Hint:* Since the DCT is unitary, you can use Parseval's relation to compute the distortion directly from the DCT coefficients.)