GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 6258
Digital Image Processing
Quiz #2

Monday, November 3, 2003

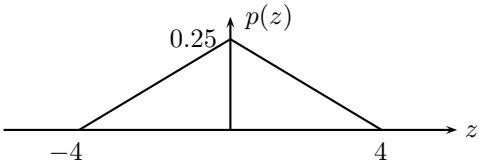
Name: _____

GENERAL INSTRUCTIONS

1. This is a *open book, open notes* exam.
2. Please do all of your work on the exam itself. You may use the backs of the pages, if necessary.
3. Please be as neat and well organized as possible.
4. Clearly indicate your answers.

Problem	*Max*	*Score*
1	30	
2	40	
3	30	
Total	100	

Problem Q2.1: We wish to design an optimal two-level quantizer (minimum mean-squared quantization error) for a sequence of independent random variables that have the probability density function shown below.



(a) Determine the threshold of the quantizer.

(b) Determine the optimal reproduction values for the two levels.

Problem Q2.2: The output of a three-level quantizer, with quantized values s_0, s_1, and s_2, satisfies a first-order Markov model with the following conditional probabilities:

$$\begin{array}{lll} p(s_0|s_0) = 1/2; & p(s_1|s_0) = 1/3; & p(s_2|s_0) = 1/6 \\ p(s_0|s_1) = 1/4; & p(s_1|s_1) = 1/2; & p(s_2|s_1) = 1/4 \\ p(s_0|s_2) = 1/6; & p(s_1|s_2) = 1/3; & p(s_2|s_2) = 1/2 \end{array}$$

(a) What is the entropy of the process?

(b) Design a Huffman code that will optimally encode **pairs** of quantized values.

(c) What is the average number of bits per output sample generated by your Huffman encoder?

Problem Q2.3: We have seen in class that the 1-D DFT can be used to compute the 1-D DCT. One enterprising student claims that the DCT can also be used to compute the DFT of a symmetric sequence, such as the impulse response of an even-length linear-phase FIR filter. Let

$$h[n] = h[M-1-n]$$

be an M-point impulse response. The student proposes to compute N samples ($N > M$) of the frequency response by using the following procedure.

1. Set
$$g[n] = \begin{cases} h[n], & n = 0, 1, \ldots, M-1 \\ 0, & \text{otherwise} \end{cases}$$
2. Compute a $2N$-point DCT of $g[n]$. Call the result $G_c[k]$.
3. Set
$$H(\omega_k) = \exp(-j\frac{\pi k}{2N})G_c[k], \quad k = 0, 1, \ldots, N-1.$$

Being a creative student, she has developed a procedure that works in principle, but, following in my footsteps, she has made one error in each step.

(a) Find the errors in the above procedure.

(b) At what frequencies will this algorithm compute the frequency response?