GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

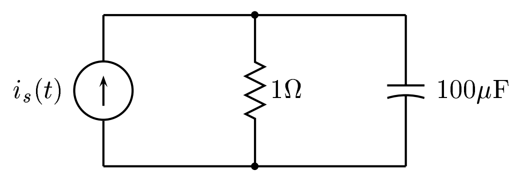
Course ECE 2040

Circuit Analysis

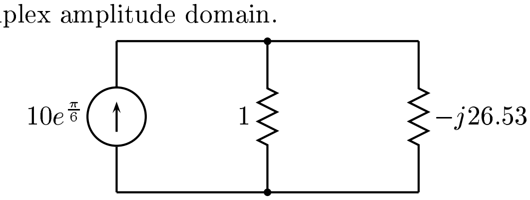
March 31, 2000

Problem Set #10–Solutions

Problem 10.1: Calculate $v(t)$ for the circuit below if $i_s(t) = 10\cos(2\pi(60)t + \pi/6)$.



Solution: While $v(t)$ isn't labeled on the figure, since the elements are all connected in parallel, it can be the voltage drop across any one of them. First, we map the circuit to the complex amplitude domain.



Note:

$$\frac{1}{j\omega C} = \frac{1}{j(120\pi)(10^{-4})} = -j26.53.$$

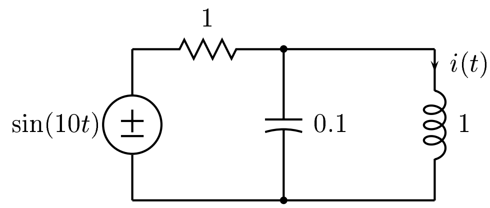
Clearly,

$$\begin{aligned} V &= Z_{eq} \cdot I_s = Z_{eq} \cdot 10\, e^{j\frac{\pi}{6}} \\ &= \frac{-j26.53}{1 - j26.53} \cdot 10\, e^{\frac{\pi}{6}} \\ &= 9.99\, e^{j(0.l477)} e^{j(1.047)} = 9.99\, e^{j(1.524)}. \end{aligned}$$

Thus,

$$v(t) = 9.99\cos(2\pi(60)t + 1.524)\ Volts$$

Problem 10.2:



Determine $i(t)$ for all t.

Solution:

$$H(j\omega) = \frac{I}{V_s} = \frac{1}{j\omega}\frac{V_\ell}{V_s} = \frac{1}{j\omega} \cdot \frac{\frac{10}{\frac{10}{j\omega}+j\omega}}{\frac{10}{\frac{10}{j\omega}+j\omega}+1} = \frac{10}{10-\omega^2+j10\omega}$$

Letting $\omega = 10$ gives

$$H(j10) = \frac{10}{10-100+j100} = \frac{1}{\sqrt{181}}e^{j\tan^{-1}\left(\frac{10}{9}-\pi\right)}$$

Therefore,

$$i(t) = -\frac{1}{\sqrt{181}}\sin(10t+\tan^{-1}(\frac{10}{9}))$$

Problem 10.3: Find $v_{out}(t)$ if $v_{in}(t) = 3 + 4\sin(1000t)$. The terminals are *open-circuited.*

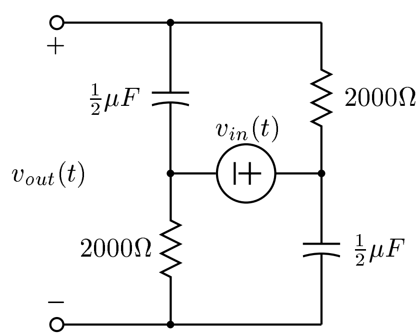
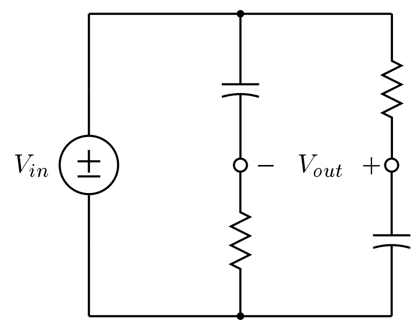


Figure 1: Circuit for Problem 10.3.

Solution: Notice that the input signal is not a sinusoid, but it is a sum of two sinusoids. Therefore we can solve the problem twice, once at a frequency $\omega = 0$ and once at a frequency $\omega = 1000$ and use superposition to get the final result. To begin we redraw this circuit as



Then

$$V_{out} = \left(\frac{\frac{1}{jC\omega}}{\frac{1}{jC\omega} + R} - \frac{R}{\frac{1}{jC\omega} + R} \right) V_{in}$$
$$= \frac{1 - jRC\omega}{1 + jRC\omega} V_{in}$$

Now, we observe

$$\omega = 0 : \quad V_{out} = V_{in}$$
$$\omega = 1000 : \quad V_{out} = \frac{1-j}{1+j} V_{in} = e^{-j\frac{\pi}{2}} V_{in}$$

Therefore,

$$v_{out}(t) = 3 + 4\sin(1000t - \frac{\pi}{2}) = 3 - 4\cos(1000t).$$

Problem 10.4: Find $v_{out}(t)$ for all t, if $v_s(t) = \cos(100t)$ for $-\infty < t < \infty$ in the circuit of Figure 2.

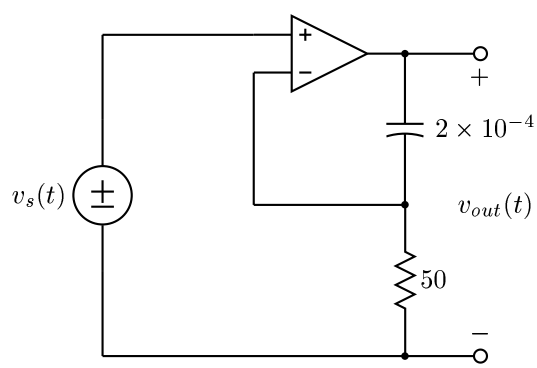


Figure 2: Circuit for Problem 10.4.

Solution: The first step is to redraw the circuit in the frequency (complex amplitude) domain. At $\omega = 100$, the capacitor has an impedance of $-j50$, the complex amplitude of the $v_s(t)$ is 1 and the complex amplitude of $v_{out}(t)$ is V_{out}. Writing a KCL equation at the node between the capacitor and resistor gives

$$\frac{1}{50} + \frac{1 - V_{out}}{-j50} = 0$$

from which we get

$$V_{out} = 1 - j = \sqrt{2}e^{-j\pi/4}$$

and

$$v_{out}(t) = \sqrt{2}\cos(100t - \frac{\pi}{4})$$

Problem 10.5: For the circuit in Figure 3 find $i(t)$ when $v_s(t) = \sin(\omega t)$.

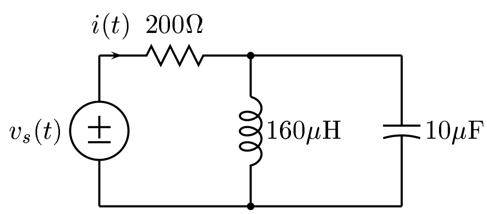


Figure 3: Circuit for Problem 10.5.

Solution: For this circuit $i(t) = \Im\text{m}(Y_{eq}e^{j\omega t})$. We can find the equivalent admittance by first finding the equivalent impedance.

$$Z_{eq} = R + \frac{jL\omega}{1 - LC\omega^2} = \frac{R - RLC\omega^2 + jL\omega}{1 - LC\omega^2}$$

Plugging in the component values, this is equal to

$$Z_{eq} = \frac{(3.2 \times 10^{-7})\omega^2 + 200 + j(1.6 \times 10^{-4})\omega}{1 - (1.6 \times 10^{-9})\omega^2}$$

Thus

$$Y_{eq} = \frac{1 - (1.6 \times 10^{-9})\omega^2}{-(3.2 \times 10^{-7})\omega^2 + 200 + j(1.6 \times 10^{-4})\omega}$$

and

$$i(t) = |Y_{eq}|\sin(\omega t - \tan^{-1}\left(\frac{(1.6 \times 10^{-4})\omega}{200 - (3.2 \times 10^{-7})\omega^2}\right)$$

The magnitude of the admittance is given by

$$|Y_{eq}| = \frac{|1 - (1.6 \times 10^{-9})\omega^2|}{(200 - (3.2 \times 10^{-7})\omega^2)^2 + (1.6 \times 10^{-4}\omega)^2}$$
