GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

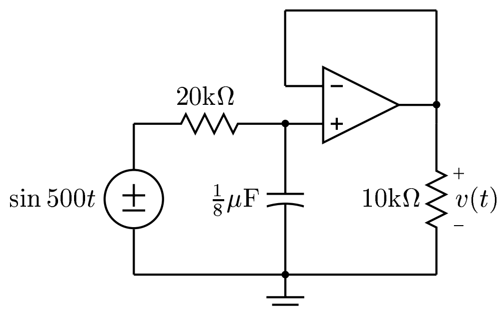
Course ECE 2040

Circuit Analysis

April 7, 2000

Problem Set #11–Solutions

Problem 11.1: For the circuit below find $v(t)$.



Solution: The opamp acts like a voltage follower, i.e., $v(t)$ is equal to the voltage across the capacitor. Furthermore, we know that $v(t)$ is of the form

$$v(t) = \Im\text{m}\left[Ve^{j500t}\right].$$

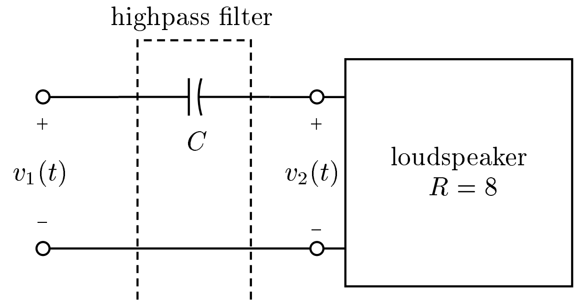
To determine V, replace the source by a complex exponential time function, the elements by their equivalent impedances, and use the voltage divider.

$$V = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}.$$

Letting $R = 20 \times 10^3$; $C = 0.125 \times 10^{-6}$, $\omega = 500$, we get

$$\begin{aligned} V &= \frac{1}{1 + j(20 \times 10^3)(0.125 \times 10^{-6})(500)} = \frac{1}{1 + j(1.25)} \\ &= 0.625e^{-j\tan^{-1} 1.25} \\ v(t) &= 0.625 \sin(500t - \tan^{-1} 1.25). \end{aligned}$$

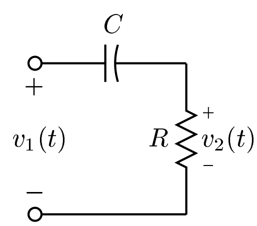
Problem 11.2: Many loudspeaker systems consist of two loudspeakers: the woofer, which reproduces the low frequency part of the signal, and the tweeter, which reproduces the high frequency part of the signal. A crossover network is used to select the high frequency part of the signal and feed it into the tweeter. Such a network functions as a highpass filter. The entire audio signal is applied at the terminals $a - a'$.



(a) Assuming that the equivalent circuit for the tweeter consists of just a resistor with a resistance of R, plot the pole-zero pattern of the system function that relates $v_2(t)$ to $v_1(t)$ and sketch the frequency response curves (magnitude and angle).

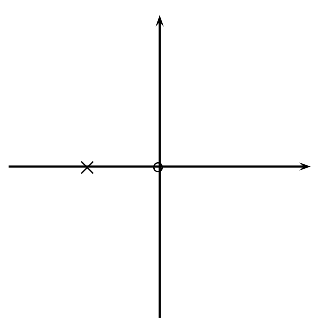
(b) If $R = 8\Omega$, find the value of the capacitance C so that the half-power frequency of the highpass filter is 5 kHz ($= 2\pi(5000)$ rad/s).

Solution:

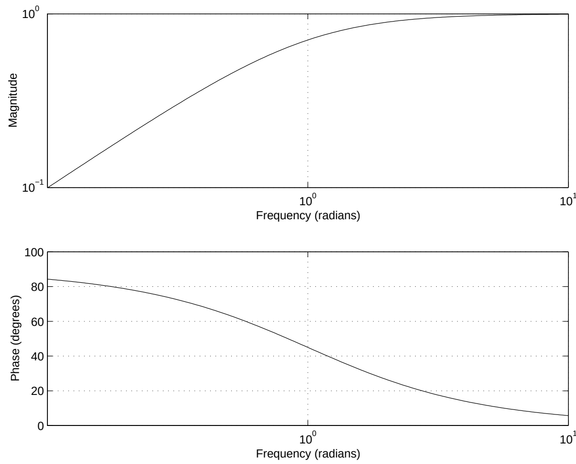


$$H(s) = \frac{R}{R + \frac{1}{Cs}} = \frac{s}{s + \frac{1}{RC}}$$

(a)



Using the MATLAB function **freqs** again gives the following plot (for positive frequencies) if $RC = 1$.



(b)

$$\frac{1}{RC} = 2\pi(5000)$$

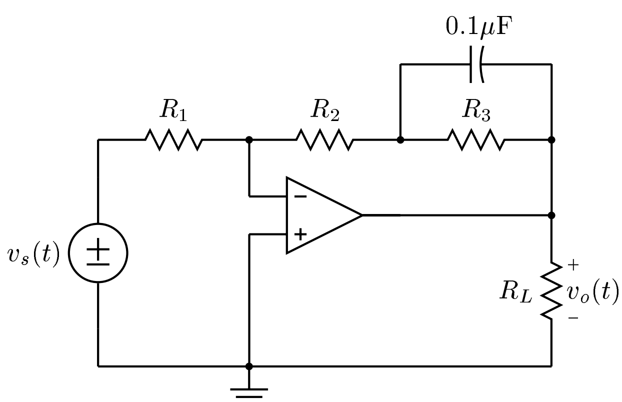
Thus,

$$C = \frac{1}{80,000\pi} \approx 4\mu F$$

Problem 11.3: In the circuit below the value of R_1 is 10kΩ.

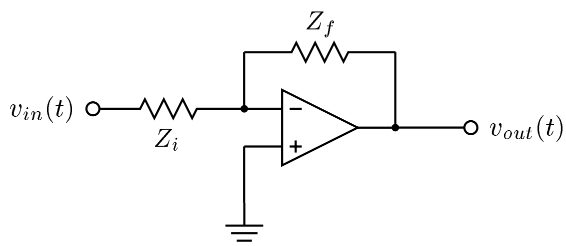
(a) Determine the values of R_2 and R_3 so that the gain (magnitude of the frequency response) at low frequencies is 5 and the gain at high frequencies is 2.

(b) Determine the frequency at which the gain is midway between these two values, i.e. the frequency at which the gain is 3.5.



Solution:

(a) This circuit is a special case of an inverting amplifier as shown below.



The frequency response of such a system is known to be

$$H(j\omega) = -\frac{Z_f(j\omega)}{Z_i(j\omega)}.$$

For this circuit, the input impedance is simply

$$Z_i = R_1$$

and the other three elements contribute to the feedback impedance

$$\begin{aligned} Z_f &= R_2 + \frac{R_3 \frac{1}{j\omega C}}{R_3 + \frac{1}{j\omega C}} = R_2 + \frac{R_3}{1 + j\omega R_3 C} \\ &= \frac{(R_2 + R_3) + j\omega R_2 R_3 C}{1 + j\omega R_3 C}. \end{aligned}$$

Therefore,

$$H(j\omega) = -\frac{(R_2 + R_3) + j\omega R_2 R_3 C}{R_1(1 + j\omega R_3 C)}.$$

To measure the low frequency gain, we can let $\omega = 0$ and take the magnitude

$$G_{LF} = \frac{R_2 + R_3}{R_1}.$$

To measure the high frequency gain, we can take the (magnitude of the) limit as ω grows large

$$G_{HF} = \frac{R_2}{R_1}.$$

Since $R_1 = 10\text{k}\Omega$ and we want $G_{LF} = 5$ and $G_{HF} = 2$, we can substitute and solve for R_2 and R_3.

$$\begin{aligned} 5 &= \frac{R_2 + R_3}{10k\Omega} \Longrightarrow R_2 + R_3 = 50k\Omega \\ 2 &= \frac{R_2}{10k\Omega} \Longrightarrow R_2 = 20k\Omega \end{aligned}$$

Therefore, we want to choose $R_2 = 20k\Omega$ and $R_3 = 30k\Omega$.

(b) With the values substituted, the frequency response is

$$H(j\omega) = -\frac{5 + j\omega(0.6)}{1 + j\omega(0.3)}.$$

To find the value of ω for which the magnitude is 3.5, we solve the equation

$$\frac{25 + \omega^2(0.6)^2}{1 + \omega^2(0.3)^2} = (3.5)^2.$$

This gives $\omega = 4.14$.

Problem 11.4: A circuit has the system function

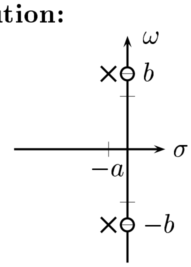
$$H(s) = \frac{s^2 + b^2}{(s+a)^2 + b^2}$$

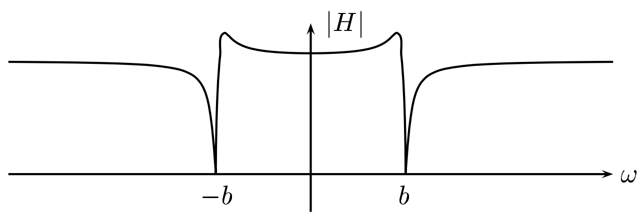
where $b >> a$.

(a) Draw the pole-zero plot. Clearly label the locations of the poles and zeros on your plots.

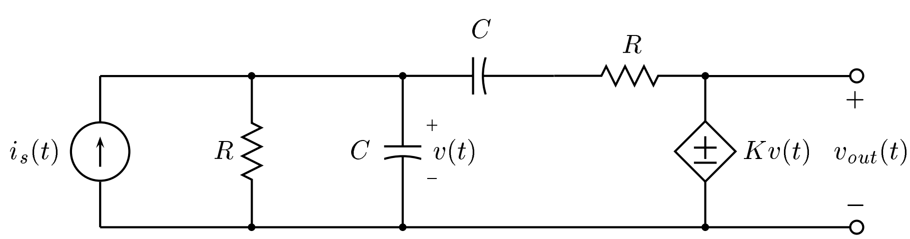
(b) Sketch the magnitude of the frequency response of the filter.

Solution:

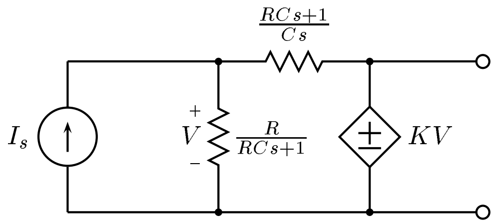




Problem 11.5: An oscillator is a circuit whose system function has a pair of complex conjugate poles located on the $j\omega$-axis in the s-plane. It is capable of supporting a sustained oscillation, i.e., once started it will continue to produce a sinusoidal output $v_{out}(t)$. In the network below determine the value of K for which the system will oscillate and determine the frequency of oscillation.



Solution: To begin we can simplify the circuit in the Laplace domain by replacing the elements in series and in parallel by their equivalent impedances. This gives the circuit shown below.



Since $V_{out} = KV$, this is not included on the figure. Let the mesh current in the right mesh be $I(s)$ (and the mesh current in the left mesh be $I_s(s)$). Then

$$(I(s) - I_s(s))\frac{R}{RCs+1} + I(s)\left(\frac{RCs+1}{Cs}\right) - K(I(s) - I_s(s))\frac{R}{RCs+1} = 0$$

We can solve this equation for the system function $H(s) = \frac{I(s)}{I_s(s)}$.

$$H(s) = \frac{(1-K)RCs}{(1-K)RCs + R^2C^2s^2 + 2RCs + 1} = \frac{\frac{1-K}{RC}s}{s^2 + \frac{3-K}{RC}s + \frac{1}{R^2C^2}}$$

From this we see that the poles will lie on the imaginary axis when $K = 3$. When $K = 3$, the poles lie at $\pm\frac{1}{RC}$, so the frequency of oscillation is $\omega_0 = \frac{1}{RC}$.
