GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

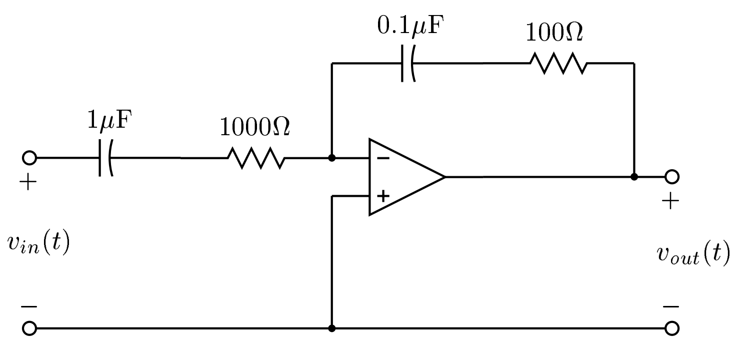
Course ECE 2040

Circuit Analysis

April 14, 2000

Problem Set #12–Solutions

Problem 12.1:

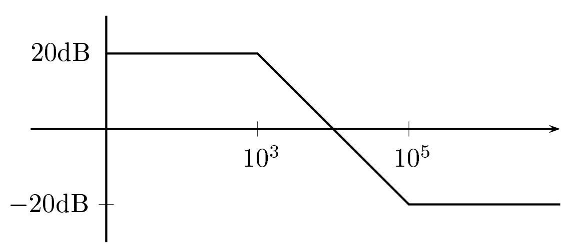


Plot the asymptotic Bode plot for the magnitude of the frequency response of the above circuit.

Solution:

$$H(s) = -\frac{100 + \frac{1}{10^{-7}s}}{1000 + \frac{1}{10^{-6}s}}$$
$$= -\frac{1}{10} \cdot \frac{s + 10^5}{s + 10^3}$$

$$H(j\omega) = \frac{1}{10} \cdot \frac{j\omega + 10^5}{j\omega + 10^3} = -10\frac{1 + j10^{-5}\omega}{1 + j10^{-3}\omega}$$



Problem 12.2: Sketch Bode magnitude and phase plots for the following system functions

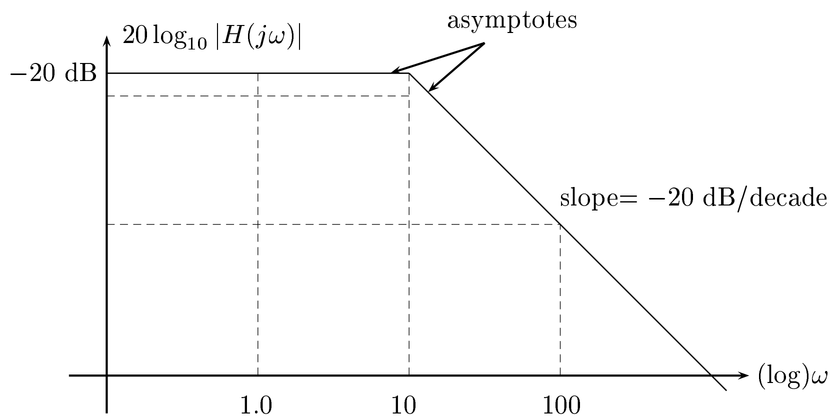
(a) $H(s) = \frac{1}{s+10}$

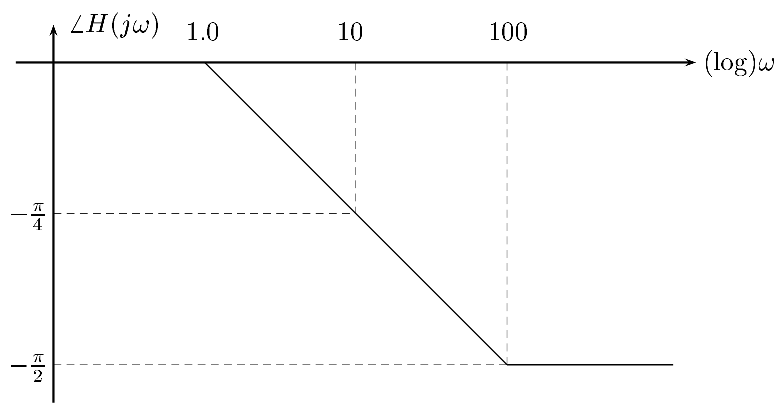
(b) $H(s) = 1 - 10s$

(c) $H(s) = \frac{s-20}{s+200}$

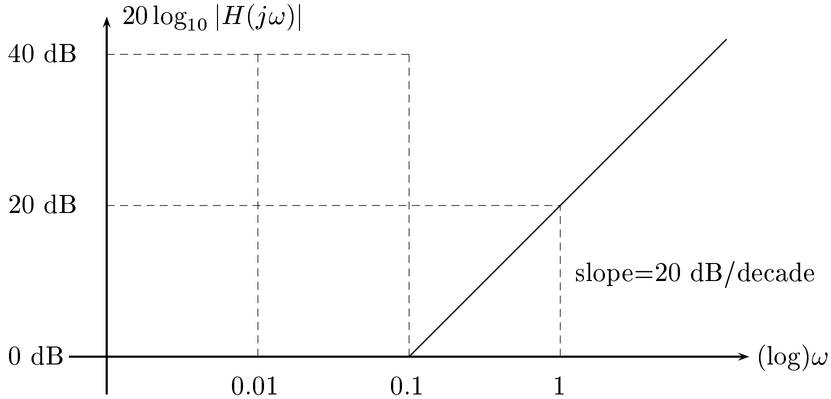
Solution:

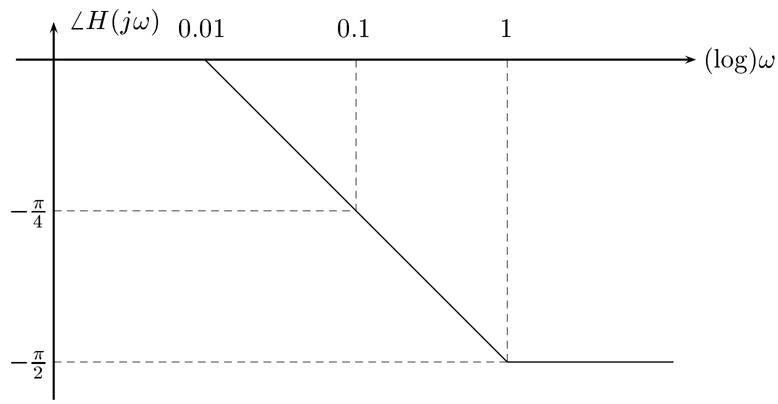
(a) For this the gain is 0.1 (= −20 dB) for low frequencies and rolls off at 20 dB/decade for high frequencies.



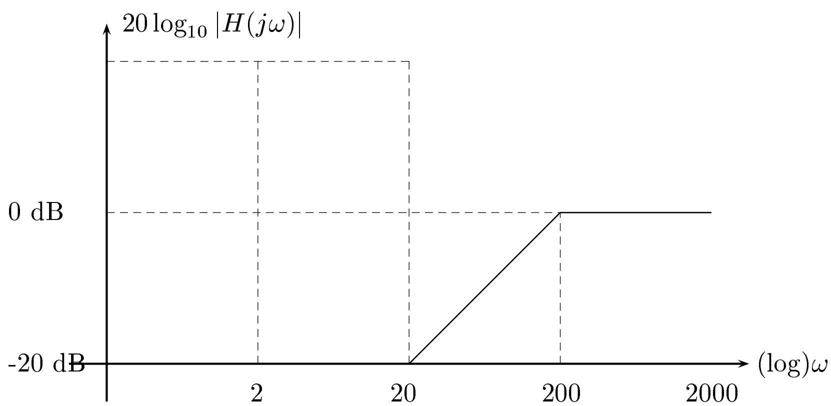


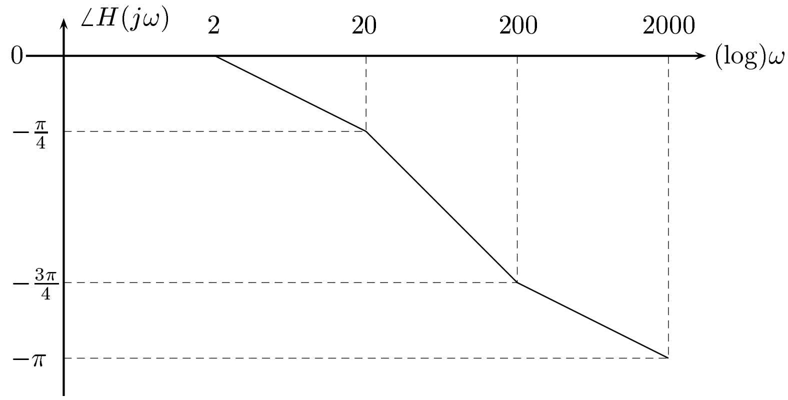
(b) Here the break frequency is 0.1 rad/s and for low frequencies the magnitude gain in 0 dB. The phase is zero at low frequencies and $-\frac{\pi}{2}$ for high ones.





(c) At frequencies below 20 the gain is constant at 0.1 (=-20 dB). At frequencies above 200 it is constant at 1.0. The phase is $-\pi$ at low frequencies and 0 at high ones.

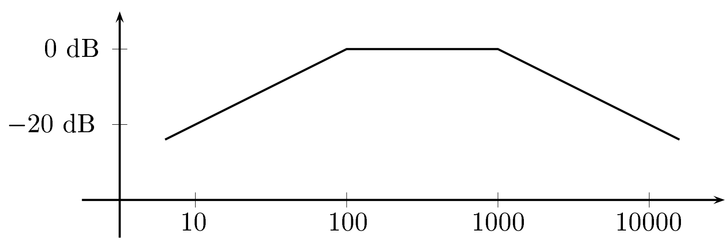




Problem 12.3: A circuit with the system function

$$H(s) = \frac{ks}{(s+a)(s+b)}$$

has the Bode magnitude plot shown below.



Determine the values of k, a, and b.

Solution: This system function consists of four components: a constant term, a zero at the origin, and two real poles. The Bode magnitude plot is, therefore, the sum of four components. The zero at the origin accounts for the low frequency increase in the frequency response. The change in the slope from 20 dB/decade to 0 dB/decade is caused by a pole at 100 rad/second. The second change in slope from 0 dB/decade to −20 dB/decade is caused by the other. Therefore

$$a = 100$$
$$b = 1000 \text{ or vice versa.}$$

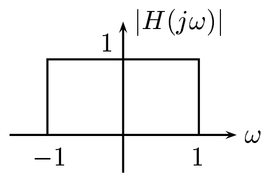
To determine the gain, we know that the gain at 100 radians/second is approximately $-6db$.

$$|H(j100)| = \frac{1}{\sqrt{2}} = \frac{k100}{|j100+100| \cdot |j100+1000|}$$

Therefore,

$$k = 1000.$$

Problem 12.4:



A circuit with the system function $H(s)$ has the approximate frequency response shown above. This system is used as the basis for a new filter whose system function, $G(s)$, is given by

$$G(s) = H(\frac{5}{s}).$$

Sketch the frequency response $G(j\omega)$.

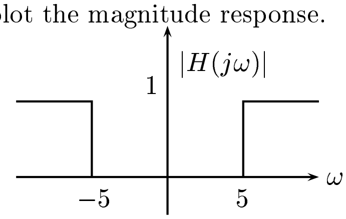
Solution: We haven't seen a problem just like this one, but we should be able to figure it out.

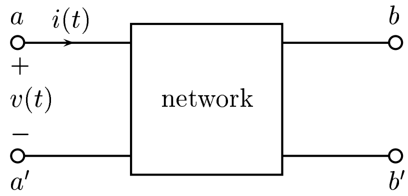
$$G(j\omega) = H(-j\frac{5}{\omega}).$$

Let's examine a few values of ω.

$$\omega = 0: \quad G(j0) = H(-j\infty) = 0$$
$$\omega = \infty: \quad G(j\infty) = H(-j0) = 1$$
$$\omega = 5: \quad G(j5) = H(-j1) = discontinuity$$

This should be enough to plot the magnitude response.





For the two-terminal pair circuit shown above the relation between $v(t)$ and $i(t)$ is

$$v(t) = 3i(t) + 2\frac{di(t)}{dt}$$

when the terminals b and b' are open-circuited and

$$v(t) = i(t)$$

when the terminals b and b' are short-circuited. Determine a possible circuit having these properties.

Solution: The answer to this problem is not unique. Here is one possibility