GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

January 21, 2000

Problem Set #1—Solutions

Problem 1.1: The voltage source in the circuit shown in Figure 1a has the source waveform shown in Figure 1b.

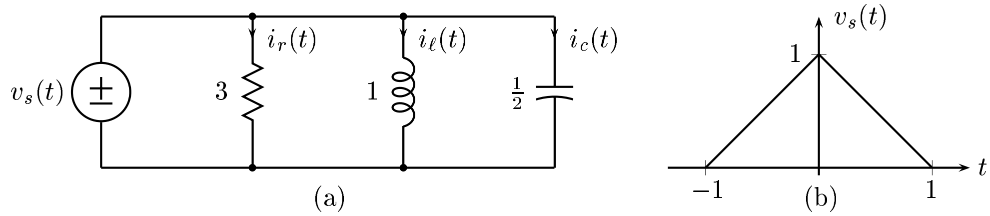


Figure 1: Circuit for Problem 1.1.

(a) Sketch $i_r(t)$.

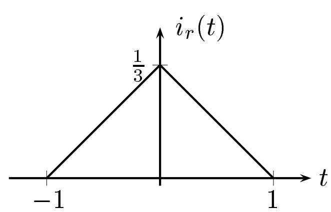
(b) Sketch $i_\ell(t)$.

(c) Sketch $i_c(t)$.

Solution:

(a)

$$i_r(t) = \frac{1}{R} v_r(t) = \frac{1}{3} v_r(t)$$



(b)

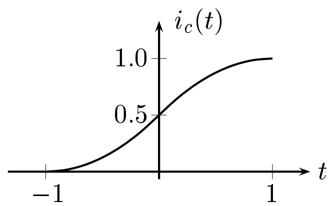
$$i_\ell(t) = \frac{1}{L} \int_{-\infty}^{t} v_\ell(\tau)\, d\tau = \int_{-\infty}^{t} v_\ell(\tau)\, d\tau$$

If we assume that $i_\ell(\infty) = 0$, then there are four cases to consider

$$i_\ell(t) = \begin{cases} 0, & t < -1 \\ \int_{-1}^{t} (\tau + 1)\, d\tau, & -1 < t < 0 \\ \frac{1}{2} + \int_{0}^{\tau} (1 - \tau)\, d\tau, & 0 < t < 1 \\ 1, & 1 < t. \end{cases}$$

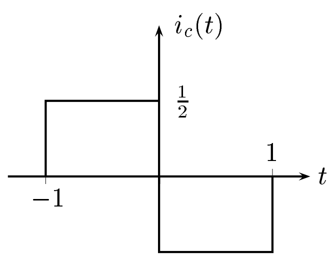
Evaluating the integrals gives

$$i_\ell(t) = \begin{cases} 0, & t < -1 \\ \frac{t^2}{2} + t + \frac{1}{2}, & -1 < t < 0 \\ -\frac{t^2}{2} + t + \frac{1}{2}, & 0 < t < 1 \\ 1, & 1 < t. \end{cases}$$



(c)

$$i_c(t) = C\frac{dv_s(t)}{dt} = \frac{1}{2}\frac{dv_s(t)}{dt} = \begin{cases} \frac{1}{2}, & -1 < t < 0 \\ -\frac{1}{2}, & 0 < t < 1 \\ 0, & \text{otherwise.} \end{cases}$$



Problem 1.2: In Figure 2 the time dependence of the element voltages and currents has been suppressed to limit clutter.

(a) Write the KCL equations that constrain the currents at all of the nodes of the network in Figure 2.

(b) Write the KVL equations that constrain the voltages for all of the meshes in that same figure.

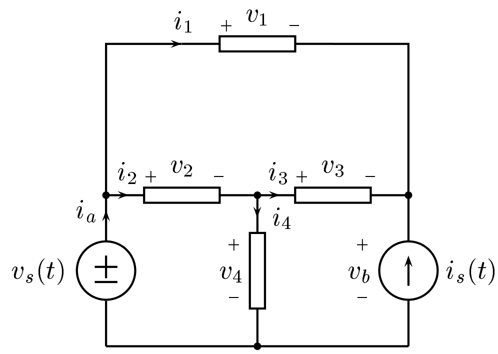


Figure 2: Figure for Problem 1.2.

Solution:

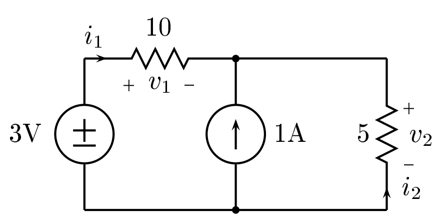
(a) There are four nodes in this circuit: upper left (a), upper middle (b), upper right (c), and lower (d).

$$\begin{aligned}
\text{node } a\text{:} \quad & i_a(t) - i_2(t) - i_1(t) = 0 \\
\text{node } b\text{:} \quad & i_2(t) - i_3(t) - i_4(t) = 0 \\
\text{node } c\text{:} \quad & i_3(t) + i_s(t) + i_1(t) = 0 \\
\text{node } d\text{:} \quad & -i_a(t) + i_4(t) - i_s(t) = 0
\end{aligned}$$

(b) There are three meshes: upper (1), lower left (2), and lower right (3).

$$\begin{aligned}
\text{mesh 1:} \quad & -v_s(t) + v_2(t) + v_4(t) = 0 \\
\text{mesh 2:} \quad & -v_4(t) + v_3(t) + v_b(t) = 0 \\
\text{mesh 3:} \quad & v_1(t) - v_3(t) - v_2(t) = 0
\end{aligned}$$

Problem 1.3: In the circuit below both source waveforms (and all of the element variables) are constant. Compute the values of i_1, v_1, i_2, and v_2.



Solution: Observe that the voltage drop across the current source is v_2 and that the current through the voltage source is i_1. This means that we do not need one KCL equation and one KVL equation. We *will* need two element relations, one KCL equation, and one KVL equation.

$$\begin{aligned} R_1: &\quad v_1 = 10 i_1 \\ R_2: &\quad v_2 = -5 i_2 \qquad \text{Notice the sign!} \\ \text{KCL:} &\quad i_1 + 1 + i_2 = 0 \\ \text{KVL:} &\quad -3 + v_1 + v_2 = 0 \end{aligned}$$

The solution of these equations is

$$\begin{aligned} i_1 &= -2/15 \qquad & v_1 &= -4/3 \\ i_2 &= -13/15 \qquad & v_2 &= 13/3. \end{aligned}$$

Problem 1.4: Two resistors connected in series act like a single resistor. Similarly, two resistors connected in parallel behave like a single resistor. In this problem, we derive these basic results.

(a) Consider two resistors connected in series and connected across a voltage source, as in Figure 3a. Show that the current flowing through them is proportional to the source voltage.

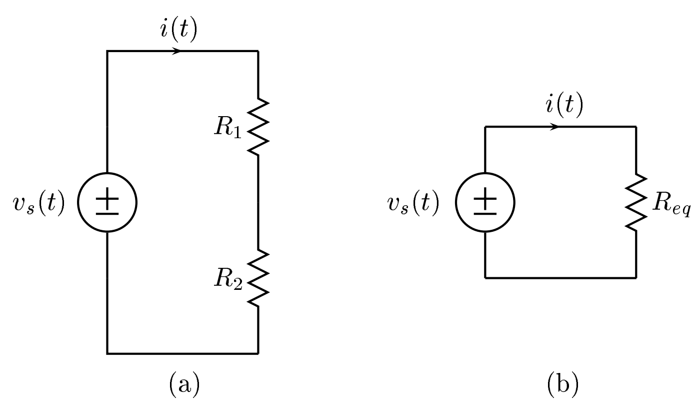


Figure 3: Two resistors in series and their equivalent resistance.

(b) This implies that from the point-of-view of the voltage source, the series connection of resistors is equivalent to a single resistor as shown in Figure 3b. Express R_{eq} in terms of R_1 and R_2.

(c) We can similarly consider two resistors connected in parallel across a voltage source, as in Figure 4a. Show again that the current flowing through them is proportional to the source voltage.

(d) Find the equivalent resistance of the parallel combination as you did in part (b).

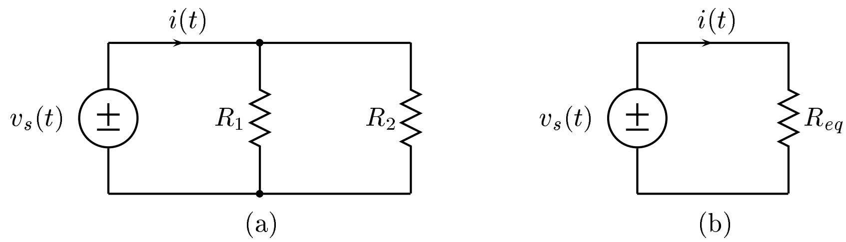


Figure 4: Two resistors in parallel and their equivalent resistance.

Solution:
(a) From KVL

$$\begin{aligned} v_s(t) &= v_1(t) + v_2(t) \\ &= R_1 i(t) + R_2 i(t) \\ &= (R_1 + R_2) i(t). \end{aligned}$$

or

$$i(t) = \frac{1}{R_1 + R_2} v_s(t).$$

(b) From the equivalent circuit

$$i(t) = \frac{1}{R_{eq}} v_s(t).$$

Therefore,

$$R_{eq} = R_1 + R_2$$

(c) From KCL

$$\begin{aligned} i(t) &= i_1(t) + i_2(t) \\ &= \frac{v_s(t)}{R_1} + \frac{v_s(t)}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_s(t). \end{aligned}$$

(d)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

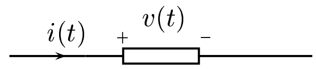
or

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Problem 1.5: The instantaneous power dissipated by an element or source is equal to the product of the voltage drop across the element, $v(t)$ and the current passing through it, $i(t)$

$$P_{inst}(t) = v(t) i(t).$$

These variables are defined as illustrated below.



The reference directions are important. Notice that the reference direction for the current is defined such that a positive value of current represents a flow from the positive terminal of the element to the negative terminal. The instantaneous power associated with an element can be either positive or negative; when it is positive, the element is dissipating power and when it is negative it is supplying power. Resistors always dissipate power.

Consider the circuit shown in Figure 5.

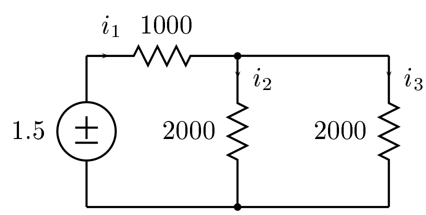


Figure 5: Circuit for Problem 1.5.

(a) This circuit contains three elements and thus there are six element variables, all of which are constant since the voltage source is constant. The currents are labeled and the voltage drops across the three resistors are implied by the default sign convention. Write a set of six linear equations in the variables v_1, v_2, v_3, i_1, i_2, and i_3 that specify the equilibrium solution. These should take the form of three element relations, one KCL equation, and two KVL equations.

(b) Solve the above set of equations to determine the equilibrium values of the element variables.

(c) Evaluate the power dissipated by all of the elements and sources. Show that the total power dissipated in the resistors is equal to the total power supplied by the source.

The net power in any circuit must always be zero, i.e. the total power dissipated must always equal the total power supplied.

Solution:

(a)

$$\begin{aligned} R_1 : \quad & v_1 = 1000 i_1 \\ R_2 : \quad & v_2 = 2000 i_2 \\ R_3 : \quad & v_3 = 2000 i_3 \\ KCL : \quad & i_1 - i_2 - i_3 = 0 \\ KVL_1 : \quad & v_1 + v_2 = 1.5 \\ KVL_2 : \quad & -v_2 + v_3 = 0 \end{aligned}$$

(b) The solution of these six equations in six unknowns is straightforward. The solution is:

$$v_1 = 0.75V \qquad i_1 = 0.75mA$$

$$v_2 = 0.75V \qquad i_2 = 0.375mA$$
$$v_3 = 0.75V \qquad i_3 = 0.375mA$$

(c)

$$P_1 = (0.75V)(0.75mA) = 0.5625mW$$
$$P_2 = (0.75V)(0.375mA) = 0.28125mW$$
$$P_3 = (0.75V)(0.375mA) = 0.28125mW$$

Therefore, the total power dissipated is:

$$P_d = P_1 + P_2 + P_3 = 1.125mW.$$

The total power supplied by the battery is

$$P_s = (1.5V)(0.75mA) = 1.125mW.$$
