GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

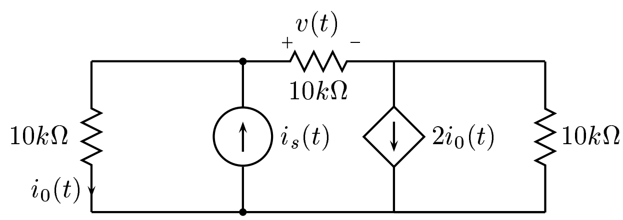
Course ECE 2040

Circuit Analysis

January 28, 2000

Problem Set #2—Solutions

Problem 2.1: Find $v(t)$ in the following circuit. You may use MATLAB to help solve the equations if you wish.



Solution: Let $v_0(t)$ be the voltage drop across the 10kΩ resistor on the left and $v_1(t)$ be the voltage drop across the 10kΩ resistor on the right, with the + terminal at the top in both cases. Let the reference directions of the three currents $i(t)$, $i_0(t)$, and $i_1(t)$ be into the + terminals of the three respective resistors.

There are six element variables and the basic network has three nodes and one mesh. Therefore, we will need to write three element relations, one KVL equation, and two KCL equations.

$$\begin{aligned}
R:&\quad v(t) = 10^4 i(t)\\
R_0:&\quad v_0(t) = 10^4 i_0(t)\\
R_1:&\quad v_1(t) = 10^4 i_1(t)\\
KCL1:&\quad i_0(t) + i(t) = i_s(t)\\
KCL2:&\quad 2i_0(t) + i_1(t) - i(t) = 0\\
KVL:&\quad -v_0(t) + v(t) + v_1(t) = 0
\end{aligned}$$

We can simplify these slightly by substituting the element relations into the KCL equations, leaving the three voltages as unknowns. This leaves three equations in three unknowns.

$$\begin{aligned}
KCL1:&\quad v_0(t) + v(t) = 10^4 i_s(t)\\
KCL2:&\quad 2v_0(t) + v_1(t) - v(t) = 0\\
KVL:&\quad -v_0(t) + v(t) + v_1(t) = 0
\end{aligned}$$

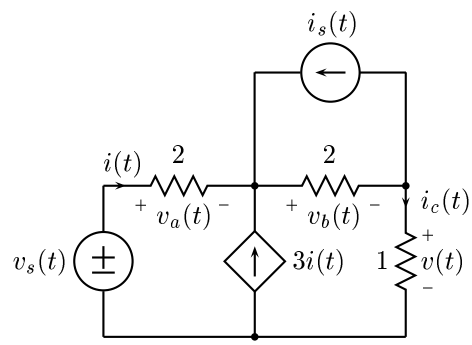
To use MATLAB for the solution, we first put these into matrix-vector form

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ v_0(t) \\ v_1(t) \end{bmatrix} = \begin{bmatrix} 10000 \\ 0 \\ 0 \end{bmatrix} i_s(t).$$

Solving gives

$$v(t) = 6000 i_s(t).$$

Problem 2.2: In the circuit below determine $v(t)$ in terms of $v_s(t)$ and $i_s(t)$.



Solution: Notice that some additional variables have been incorporated into the circuit diagram for the problem. Since there are six element variables, we will need a total of six equations—three element relations, one KVL equation, and two KCL equations.

$$\begin{aligned}
R_a: &\quad v_a(t) = 2i_a(t)\\
R_b: &\quad v_b(t) = 2i_b(t)\\
R_c: &\quad v_c(t) = i_c(t)\\
KVL: &\quad v_a(t) + v_b(t) + v(t) = v_s(t)\\
KCL1: &\quad i(t) + 3i(t) - i_b(t) = -i_s(t)\\
KCL2: &\quad -i_c(t) + i_b(t) = i_s(t)
\end{aligned}$$

Inserting the element relations into the KCL equations permits the number of unknowns and the number of equations to be reduced.

$$\begin{aligned}
v(t) + v_a(t) + v_b(t) &= v_s(t)\\
8v_a(t) - v_b(t) &= -si_s(t)\\
-2v(t) + v_b(t) &= 2i_s(t)
\end{aligned}$$

Putting these into matrix-vector form

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 8 & 1 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ v_a(t) \\ v_b(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_s(t) + \begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix} i_s(t).$$

Then

$$v(t) = 0.3077 v_s(t) - 0.6154 i_s(t).$$

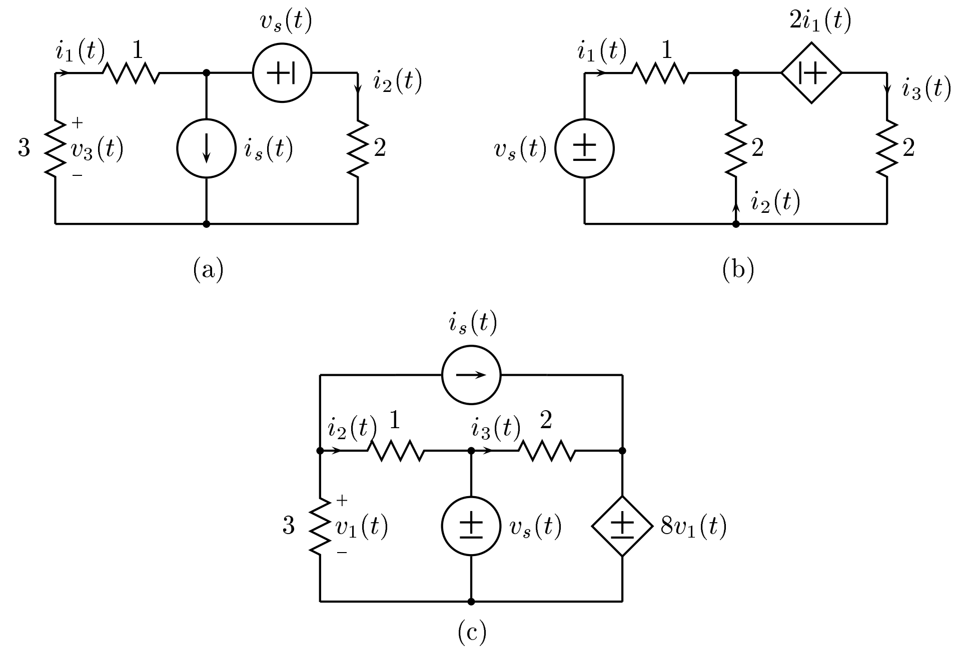


Figure 1: Networks for Problem 2.3.

Problem 2.3: This problem is concerned with the three networks shown in Figure 1.

(a) For the network in (a)

(i.) Draw the basic network.

(ii.) Identify the closed paths in the original network that correspond to meshes in the basic network.

(iii.) Identify the closed surfaces in the original network that correspond to nodes in the basic network.

(b) Repeat for the network in (b).

(c) Repeat for the network in (c).

Solution:

(a) (i) The basic network is shown in Figure 2.

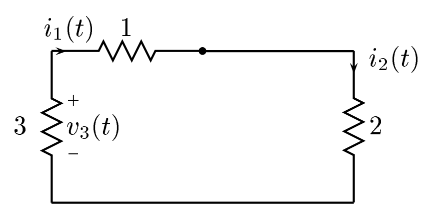


Figure 2: The basic network for the circuit in Figure 1(a).

(ii) There is only one mesh in the basic network and it corresponds to the path around the outside of the original network, i.e. it is the path that contains both the 3Ω and the 2Ω resistors.

(iii) The surfaces in the complete network that correspond to nodes in the basic network are indicated in Figure 3 as dashed lines.

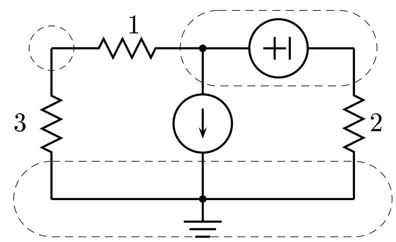


Figure 3: The surfaces in the complete network in Figure 1(a) corresponding to nodes in the basic network.

(b) (i) The basic network is shown in Figure 4.

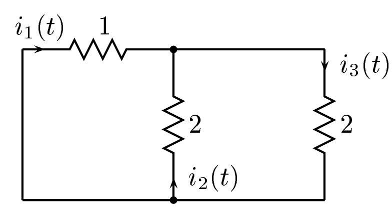


Figure 4: The basic network for the circuit in Figure 1(b).

(ii) The basic network contains two meshes. The paths corresponding to those meshes in the complete network are the two meshes in the complete network.

(iii) The basic network contains two nodes. In the complete network, those correspond to the two closed surfaces illustrated in Figure 5.

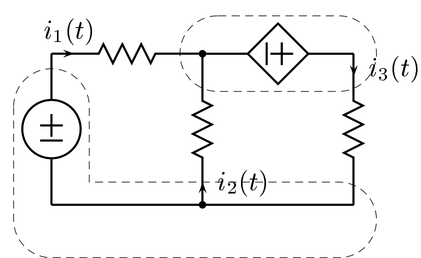


Figure 5: The surfaces in the complete network of Figure 1(b) corresponding to nodes in the basic network of Figure 4.

(c) (i) The basic network is shown in Figure 6.

(ii) The meshes in the basic network correspond to the two lower meshes in the complete network.

(iii) The basic network contains only two nodes. In the complete network these correspond to the dashed surfaces shown in Figure 7. Since both termi-

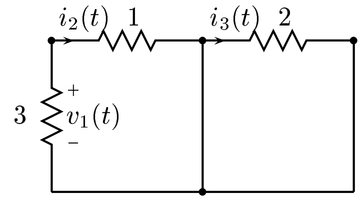


Figure 6: The basic network for the circuit in Figure 1(c).

nals of the 2Ω resistor are included in that node, the resistor itself can be incorporated in it as well.

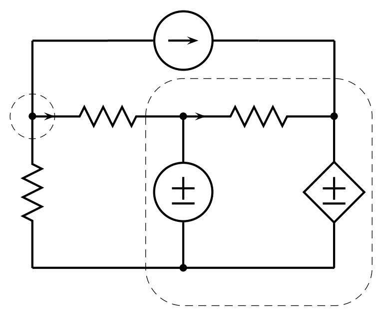
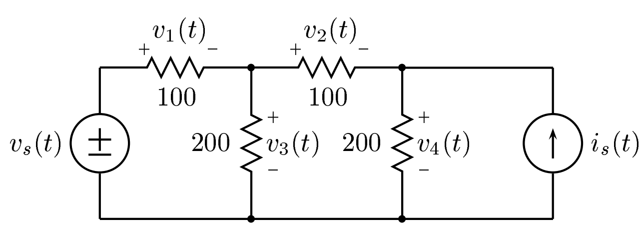


Figure 7: The surfaces for the network in Figure 1(c) corresponding to nodes in the basic network of Figure 4.

Problem 2.4:



(a) Determine the number of KCL and KVL equations that we will have to write in order to find all four voltages, $v_1(t)$, $v_2(t)$, $v_3(t)$, and $v_4(t)$.

(b) Write a complete set of linear equations that must be solved to find the equilibrium solution.

(c) Use MATLAB to find $v_1(t)$, $v_2(t)$, $v_3(t)$, and $v_4(t)$. Each voltage should be of the form

$$v_k(t) = a_k v_s(t) + b_k i_s(t).$$

Solution:

(a) Since there are two meshes and three nodes in the basic network, we will have to write two KVL equations and two KCL equations in addition to four element relations.

(b) Using the default sign conventions,

$$\begin{aligned}
& v_1(t) - 100 i_1(t) = 0 \\
& v_2(t) - 100 i_2(t) = 0 \\
& v_3(t) - 200 i_3(t) = 0 \\
& v_4(t) - 200 i_4(t) = 0 \\
\text{KCL1:}\quad & i_1(t) - i_2(t) - i_3(t) = 0 \\
\text{KCL2:}\quad & -i_2(t) + i_4(t) = i_s(t) \\
\text{KVL1:}\quad & v_1(t) + v_3(t) = v_s(t) \\
\text{KVL2:}\quad & v_2(t) - v_3(t) + v_4(t) = 0
\end{aligned}$$

(c) First we substitute the element relations

$$\begin{aligned}
& 2v_1(t) - 2v_2(t) - v_3(t) = 0 \\
& -2v_2(t) + v_4(t) = 200 i_s(t) \\
& v_1(t) + v_3(t) = v_s(t) \\
& v_2(t) - v_3(t) + v_4(t) = 0.
\end{aligned}$$

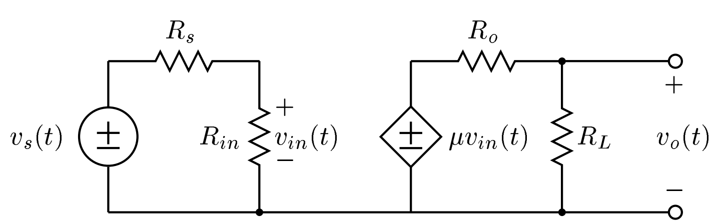
Putting these in matrix-vector form gives

$$\begin{bmatrix} 2 & -2 & -1 & 0 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} v_s(t) + \begin{bmatrix} 0 \\ 200 \\ 0 \\ 0 \end{bmatrix} i_s(t).$$

Then

$$\begin{aligned}
v_1(t) &= 0.4545 v_s(t) - 36.3636 i_s(t) \\
v_2(t) &= 0.1818 v_s(t) - 54.5455 i_s(t) \\
v_3(t) &= 0.5455 v_s(t) + 36.3636 i_s(t) \\
v_4(t) &= 0.3636 v_s(t) + 90.9091 i_s(t).
\end{aligned}$$

Problem 2.5: (Irwin and Wu, problem 2.101.) For the network below, choose the values of R_{in} and R_o such that $v_o(t)$ is maximized. What is the resulting ratio $|v_o(t)/v_s(t)|$?



Solution: From the subnetwork on the left

$$i_{in}(t) = \frac{1}{R_{in}} v_{in}(t)$$

$$-v_s(t)(t) + \frac{R_s}{R_{in}} v_{in}(t) + v_{in}(t) = 0$$

These imply

$$v_{in}(t) = \frac{1}{1 + \frac{R_s}{R_{in}}} v_s(t) = \frac{R_{in}}{R_{in} + R_s} v_s(t).$$

From the subnetwork on the right

$$-\mu v_{in}(t) + \frac{R_o}{R_L} v_o(t) + v_0(t) = 0$$

$$v_o(t) = \frac{\mu R_L R_{in}}{(R_L + R_o)(R_{in} + R_s)} v_s(t).$$

To maximize the amplitude of the output, we want to make R_o as small as possible and R_{in} as large as possible. Therefore

$$R_o = 0$$
$$R_{in} = \infty$$

and

$$\frac{v_o(t)}{v_s(t)} = \mu$$