

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040
Circuit Analysis

February 4, 2000

Problem Set #3–Solutions

Problem 3.1: We wish to solve the circuit in Figure 1 using the node method. Let $e_a(t)$ be the node potential at the indicated node.

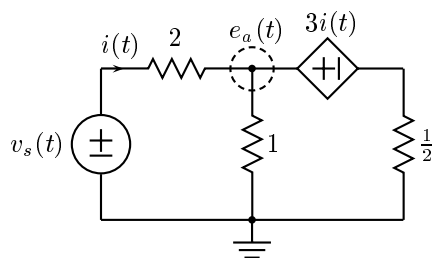


Figure 1: Circuit for Problem 3.1.

- (a) Express $i(t)$ in terms of $e_a(t)$ and $v_s(t)$.
- (b) Write a KCL equation at the surface in the complete network that corresponds to the non-ground node in the basic network. This equation should involve only the variables $e_a(t)$ and $v_s(t)$.
- (c) Determine $e_a(t)$.

Solution:

(a) $i(t) = \frac{1}{2} [v_s(t) - e_a(t)]$

- (b) This surface is a supernode that envelops the dependent voltage source. The potential at the negative terminal of the dependent source is $e_a(t) - 3i(t) = e_a(t) - \frac{3}{2}(v_s(t) - e_a(t)) = \frac{5}{2}e_a(t) - \frac{3}{2}v_s(t)$.

$$\frac{1}{2}[e_a(t) - v_s(t)] + e_a(t) + 2[\frac{5}{2}e_a(t) - \frac{3}{2}v_s(t)] = 0$$

Simplifying

$$\frac{13}{2}e_a(t) = \frac{7}{2}v_s(t).$$

(c) $e_a(t) = \frac{7}{13}v_s(t)$

Problem 3.2: Find all of the element voltages and currents in the circuit of Figure 2 using the mesh method. Be sure to identify the variables clearly.

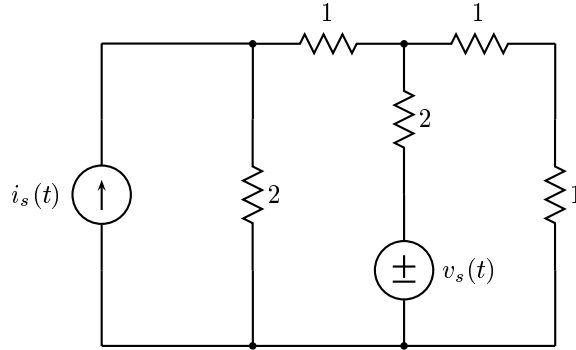


Figure 2: Circuit for Problem 3.2.

Solution: The first step is to identify the meshes in the basic network and to identify the variables in the circuit. This is done in Figure 3. The voltages are implied by the currents using the default sign convention. The two mesh equations are

$$\text{mesh } \alpha: 2[i_\alpha(t) - i_s(t)] + i_\alpha(t) + 2[i_\alpha - i_\beta] + v_s(t) = 0$$

$$\text{mesh } \beta: -v_s(t) + 2[i_\beta(t) - i_\alpha(t)] + i_\beta(t) + i_\beta(t) = 0.$$

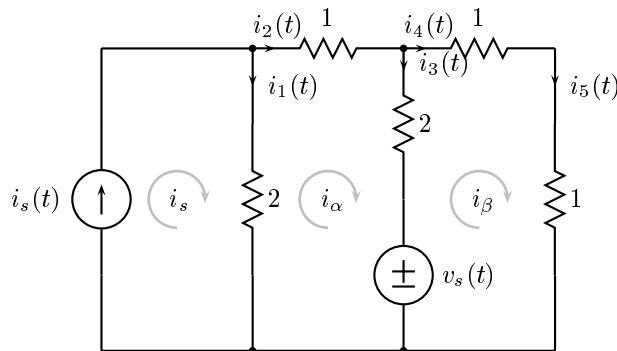


Figure 3: Circuit for Problem 3.2 with the meshes indicated and the currents defined.

These can be put into matrix-vector form as:

$$\begin{bmatrix} 5 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} i_s(t) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} v_s(t)$$

The solution is

$$\begin{aligned} i_\alpha(t) &= \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t) \\ i_\beta(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t). \end{aligned}$$

From these we can compute all of the element variables.

$$\begin{aligned} i_1(t) &= \frac{1}{2}i_s(t) + \frac{1}{8}v_s(t) \\ i_2(t) &= \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t) \\ i_3(t) &= \frac{1}{4}i_s(t) - \frac{5}{16}v_s(t) \\ i_4(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t) \\ i_5(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t) \\ v_1(t) &= i_s(t) + \frac{1}{4}v_s(t) \\ v_2(t) &= \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t) \\ v_3(t) &= \frac{1}{2}i_s(t) - \frac{5}{8}v_s(t) \\ v_4(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t) \\ v_5(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t) \end{aligned}$$

Problem 3.3: (a) Which method, the mesh method or the node method, will result in fewer equations to solve in order to determine $v(t)$ in the circuit in Figure 4?

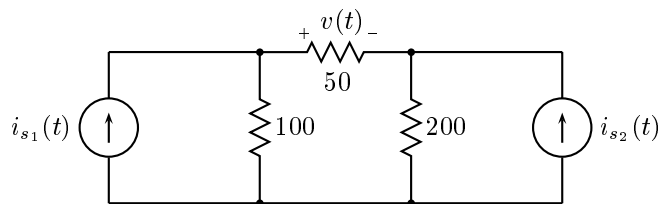


Figure 4: Circuit for Problem 3.3.

- (b) Determine $v(t)$ using the method that you selected in (a).

Solution:

- (a) The mesh method requires writing only one KVL equation, because the basic network contains only one mesh. The node method, on the other hand requires writing two KCL equations because the basic network contains three nodes. Therefore, the mesh method will result in fewer equations.
- (b) Let $i_{s_1}(t)$ be a clockwise mesh current in the left mesh of the complete circuit, $i(t)$ be a clockwise mesh current in the center mesh (the only one around which we write a KVL equation), and $i_{s_2}(t)$ be the counterclockwise mesh current around the right mesh. Then the KVL equation is

$$100[i(t) - i_{s_1}(t)] + 50i(t) + 200[i(t) + i_{s_2}(t)] = 0.$$

Solving for $i(t)$ gives

$$\begin{aligned} 350i(t) &= 100i_{s_1} - 200i_{s_2}(t) \\ i(t) &= \frac{2}{7}i_{s_1} - \frac{4}{7}i_{s_2}(t) \end{aligned}$$

From this

$$v(t) = \frac{100}{7}i_{s_1}(t) - \frac{200}{7}i_{s_2}(t).$$

Problem 3.4: The node method and the mesh method are not the only systematic methods for finding the equilibrium solution of a circuit, although they are the most popular. As an example of a different approach, consider the circuit in Figure 5.

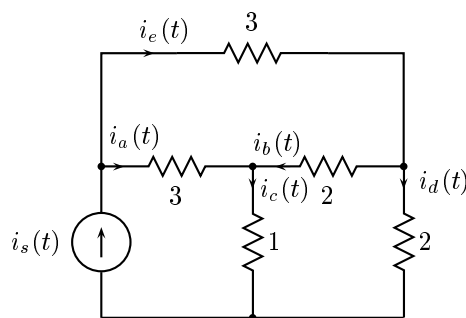


Figure 5: Circuit for Problem 3.4

- (a) Show that all of the currents in the circuit (and, therefore, all of the voltages) can be expressed in terms of $i_a(t)$, $i_b(t)$, and $i_s(t)$, i.e., express $i_c(t)$, $i_d(t)$, and $i_e(t)$ in terms of $i_a(t)$, $i_b(t)$, and $i_s(t)$.

- (b) Write a KVL equation over the path defined by each mesh in the basic network using only $i_a(t)$, $i_b(t)$, and $i_s(t)$ as variables.
- (c) Express your equations in matrix-vector form by filling in the missing entries in the equation below:

$$\begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} i_s(t)$$

- (d) Solve your equations in (c) and use your solution to derive values for all the voltages and currents in the network.

Solution:

- (a) From KCL at the center node

$$\boxed{i_c(t) = i_a(t) + i_b(t).}$$

From KCL at the bottom node

$$i_c(t) + i_d(t) = i_s(t).$$

Thus,

$$\boxed{i_d(t) = i_s(t) - i_c(t) = i_s(t) - i_a(t) - i_b(t).}$$

From KCL at the left center node

$$i_e(t) + i_a(t) = i_s(t)$$

or

$$\boxed{i_e(t) = i_s(t) - i_a(t).}$$

- (b) The basic network contains only two meshes: the upper mesh and the lower right mesh. For the upper mesh the KVL equation is

$$3i_e(t) + 2i_b(t) - 3i_a(t) = 0$$

Substituting and simplifying we get

$$\begin{aligned} 3[i_s(t) - i_a(t)] + 2i_b(t) - 3i_a(t) &= 0 \\ -6i_a(t) + 2i_b(t) &= -3i_s(t) \end{aligned}$$

For the lower right mesh

$$-i_c(t) - 2i_b(t) + 2i_d(t) = 0$$

This becomes

$$\begin{aligned} -[i_a(t) + i_b(t)] - 2i_b(t) + 2[i_s(t) - i_a(t) - i_b(t)] &= 0 \\ -3i_a(t) - 5i_b(t) &= -2i_s(t) \end{aligned}$$

(c) Multiplying both equations by -1 removes some of the minus signs.

$$\begin{bmatrix} 6 & -2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} i_s(t)$$

(d) Using MATLAB, we find

$$\begin{aligned} i_a(t) &= 0.5278 i_s(t) \\ i_b(t) &= 0.0833 i_s(t) \end{aligned}$$

We can now use the relations that we derived in part (a) to solve for the remaining currents

$$\begin{aligned} i_c(t) &= i_a(t) + i_b(t) = 0.6111 i_s(t) \\ i_d(t) &= i_s(t) - i_c(t) = 0.3889 i_s(t) \\ i_e(t) &= i_s(t) - i_a(t) = 0.4722 i_s(t). \end{aligned}$$

From the currents, we can now compute the element voltages using Ohm's Law. Definitions of these voltage drops follow our default sign convention.

$$\begin{aligned} v_a(t) &= 3i_a(t) = 1.5834 i_s(t) \\ v_b(t) &= 2i_b(t) = 0.1666 i_s(t) \\ v_c(t) &= i_c(t) = 0.6111 i_s(t) \\ v_d(t) &= 2i_d(t) = 0.7778 i_s(t) \\ v_e(t) &= 3i_e(t) = 1.4166 i_s(t). \end{aligned}$$

Finally, the voltage drop across the current source is

$$v_s(t) = v_a(t) + v_c(t) = 2.1845 i_s(t).$$

Problem 3.5: In our derivation of the mesh method, we stressed its duality with the node method, i.e., the similarity of the two methods if the roles of voltages and currents, and nodes and meshes are reversed. This problem lets you explore this issue further. Consider the network in Figure 6.

- (a) Use the node method to determine the set of equations that must be solved to find the equilibrium solution. Omit the ground node when writing your equations. Express these equations in the form

$$\mathbf{C}\mathbf{v}(t) = \mathbf{s}_1 v_{s_1}(t) + \mathbf{s}_2 v_{s_2}(t).$$

Here $\mathbf{v}(t)$ is a vector of node potentials, \mathbf{s}_1 and \mathbf{s}_2 are column vectors of constants, and \mathbf{C} is a constant matrix.

- (b) Now design a *different* network containing two *current* sources with currents $i_{s_1}(t)$ and $i_{s_2}(t)$, such that the set of *mesh* equations that need to be solved to find the equilibrium solution is

$$\mathbf{C}\mathbf{i}(t) = \mathbf{s}_1 i_{s_1}(t) + \mathbf{s}_2 i_{s_2}(t).$$

and $\mathbf{i}(t)$ is the vector of mesh currents, where \mathbf{C} , \mathbf{s}_1 and \mathbf{s}_2 are the same as for your solution in part (a).

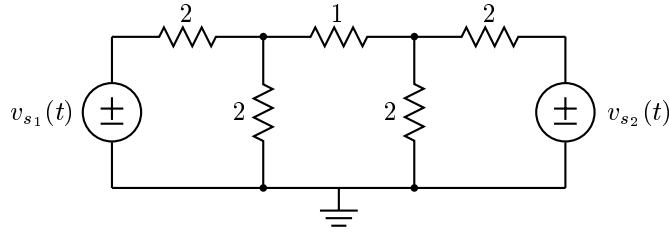


Figure 6: Circuit for Problem 3.5.

- (c) Solve your equations from part (b).

Solution:

- (a) There are two nodes, which are located at the terminals of the 1Ω resistor. These lead to the two equations

$$\text{node } a: \quad \frac{1}{2}[e_a(t) - v_{s_1}(t)] + \frac{1}{2}e_a(t) + [e_a(t) - e_b(t)] = 0$$

$$\text{node } b: \quad [e_b(t) - e_a(t)] + \frac{1}{2}e_b(t) + \frac{1}{2}[e_b(t) - v_{s_2}(t)] = 0,$$

which can be written in matrix-vector form as

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} v_{s_1}(t) + \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} v_{s_2}(t).$$

- (b) The mesh equations for the desired circuit should look like the node equations for the previous one. Thus, they should have the form

$$\text{mesh } a: \quad \frac{1}{2}[i_a(t) - i_{s_1}(t)] + \frac{1}{2}i_a(t) + [i_a(t) - i_b(t)] = 0$$

$$\text{mesh } b: \quad [i_b(t) - i_a(t)] + \frac{1}{2}i_b(t) + \frac{1}{2}[i_b(t) - i_{s_2}(t)] = 0$$

One circuit that is consistent with these mesh equations is the one drawn in Figure 7.

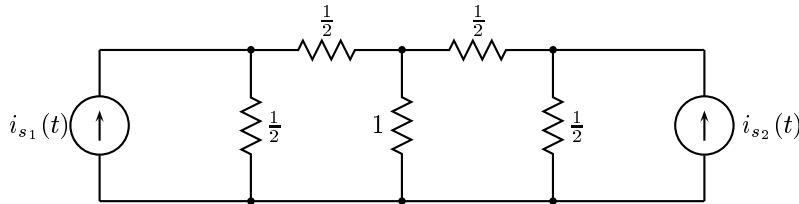


Figure 7: The dual to the circuit in Figure ??.

- (c)

$$i_a(t) = \frac{1}{3}i_{s_1}(t) + \frac{1}{6}i_{s_2}(t)$$

$$i_b(t) = \frac{1}{6}i_{s_1}(t) + \frac{1}{3}i_{s_2}(t)$$

