GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

February 11, 2000

Problem Set #4–Solutions

Problem 4.1: Find the $v - i$ relation of the two-terminal network shown in Figure 1.

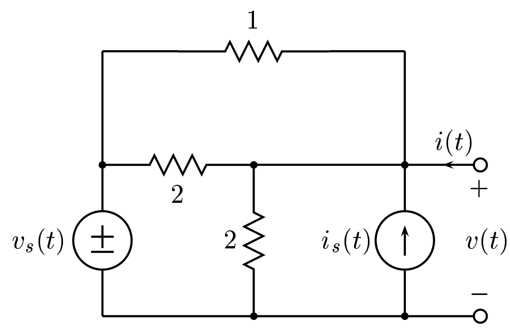


Figure 1: Two-port network for Problem 4.1.

Solution: For this circuit, we can find the $v - i$ relation by using the node method. Let the node connected to the − terminal be the ground. Then the potential at the node connected to the + terminal is $v(t)$. Writing a KCL at this node provides the relation

$$\frac{1}{2}[v(t) - v_s(t)] + \frac{1}{2}v(t) - i_s(t) - i(t) + [v(t) - v_s(t)] = 0.$$

This simplifies to

$$2v(t) = \frac{3}{2}v_s(t) + i_s(t) + i(t)$$

or

$$v(t) = \frac{1}{2}i(t) + \frac{3}{4}v_s(t) + \frac{1}{2}i_s(t)$$

which is the desired $v - i$ relation. This problem could also be solved by a number of other means including using source superposition.

Problem 4.2: You have a box containing an unlimited number of 10KΩ resistors. Show how to connect some of these together to construct equivalent resistances with the following values:

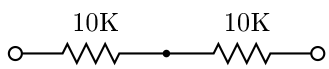
(a) 20KΩ

(b) 25KΩ

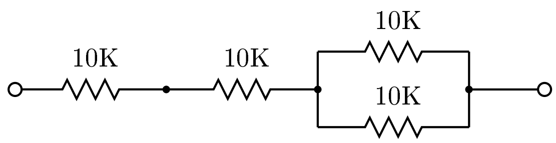
(c) 6667Ω

Solution:

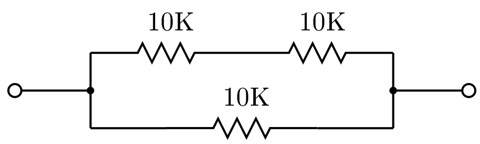
(a)



(b)



(c)



Problem 4.3: Consider a one-port network consisting of two capacitors with capacitances C_1 and C_2 connected in series, as shown in Figure 2.

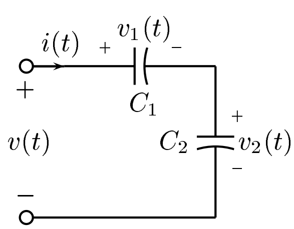


Figure 2: Two capacitors connected in series.

(a) Show that this network is equivalent to a single capacitor.

(b) Derive a formula for the equivalent capacitance C_{eq} in terms of C_1 and C_2.

(c) Derive expressions for the voltage $v_1(t)$ measured across capacitor C_1 and the voltage $v_2(t)$ measured across C_2 in terms of the voltage $v(t)$ appearing across the series connection.

Solution:

(a) We know that

$$i(t) = C_1 \frac{dv_1(t)}{dt} \Longrightarrow \frac{dv_1(t)}{dt} = \frac{i(t)}{C_1}$$
$$i(t) = C_2 \frac{dv_2(t)}{dt} \Longrightarrow \frac{dv_2(t)}{dt} = \frac{i(t)}{C_2}$$

and from KVL we know

$$v(t) = v_1(t) + v_2(t).$$

Taking a derivative of both sides gives

$$\frac{dv(t)}{dt} = \frac{dv_1(t)}{dt} + \frac{dv_2(t)}{dt} = \frac{i(t)}{C_1} + \frac{i(t)}{C_2}.$$

Therefore,

$$\frac{dv(t)}{dt} = \frac{i(t)}{C_1} + \frac{i(t)}{C_2}$$

or

$$i(t) = \left(\frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \right) \frac{dv(t)}{dt}$$

and the current is seen to be proportional to the first derivative of the voltage.

(b) From part (a) we see

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

(c) We know that

$$i(t) = \left(\frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \right) \frac{dv(t)}{dt} = C_1 \frac{dv_1(t)}{dt}.$$

Therefore,

$$\frac{dv_1(t)}{dt} = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2}} \frac{dv(t)}{dt}.$$

Integrating both sides gives the desired result:

$$v_1(t) = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2}} v(t).$$

Similarly

$$v_2(t) = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2}} v(t).$$

Problem 4.4: Two two-terminal networks, N_1 and N_2, have the $v - i$ relations

$$N_1 : \quad v(t) = R_1 i(t) + v_{T1}$$
$$N_2 : \quad v(t) = R_2 i(t) + v_{T2}.$$

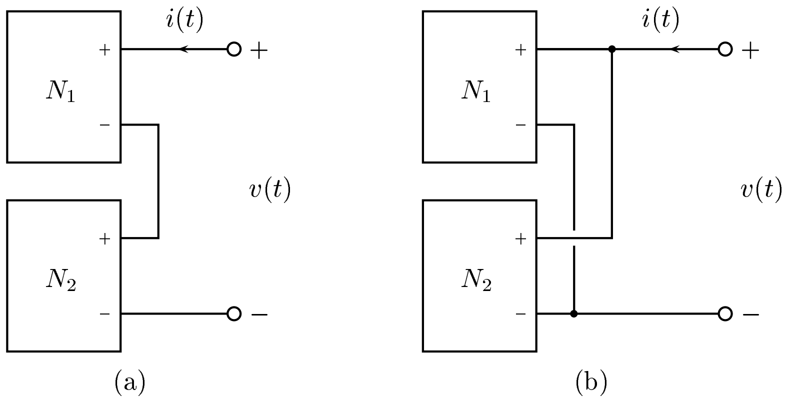


Figure 3: (a) A series connection of two two-terminal networks. (b) A parallel connection of two two-terminal networks.

(a) The two networks are connected in series if the same current passes through both, as illustrated in Figure 3a. Determine the $v-i$ characteristic for the series connection of N_1 and N_2.

(b) The two networks are connected in parallel if the same voltage appears across the terminals of both networks, as illustrated in Figure 3b. Determine the $v-i$ characteristic for the parallel connection of N_1 and N_2.

Solution:

(a) Let the voltage drop across the terminals of N_1 be $v_1(t)$ and the voltage drop across the terminals of N_2 be $v_2(t)$. The current $i(t)$ passes through both networks. Therefore, we can rewrite the two $v-i$ relations as

$$\begin{aligned} v_1(t) &= R_1 i(t) + v_{T1} \\ v_2(t) &= R_2 i(t) + v_{T2}. \end{aligned}$$

However, $v(t) = v_1(t) + v_2(t)$. Therefore, if we add the two equations,

$$\begin{aligned} v(t) &= R_1 i(t) + v_{T1} + R_2 i(t) + v_{T2} \\ &= [R_1 + R_2] i(t) + [v_{T1} + v_{T2}] \end{aligned}$$

(b) The approach here is similar except that both networks now have the same voltage drop $v(t)$ and currents $i_1(t)$ and $i_2(t)$. Therefore,

$$\begin{aligned} v(t) &= R_1 i_1(t) + v_{T1} \\ v(t) &= R_2 i_2(t) + v_{T2}. \end{aligned}$$

By KCL

$$\begin{aligned} i(t) &= i_1(t) + i_2(t) \\ &= \frac{v(t) - v_{T1}}{R_1} + \frac{v(t) - v_{T2}}{R_2} \\ &= \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v(t) - \left(\frac{1}{R_1} v_{T1} + \frac{1}{R_2} v_{T2}\right). \end{aligned}$$

This form is perfactly acceptable for the $v - i$ relation, but if it is more convenient to have $v(t)$ on the left, we can rewrite it as

$$v(t) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} i(t) + \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} v_{T_1} + \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} v_{T_2}.$$

Problem 4.5: Express $v(t)$ as a function of $i(t)$ and $i_s(t)$ for the circuit in Figure 4.

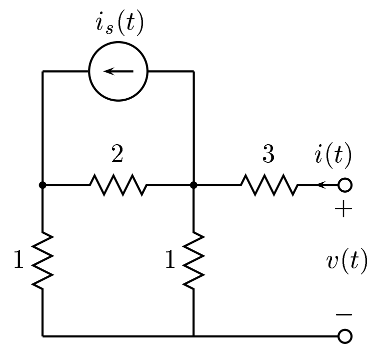
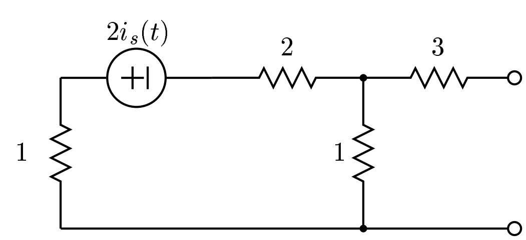
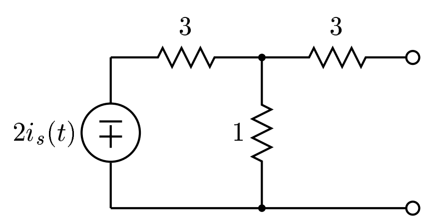


Figure 4: Circuit for Problem 4.5.

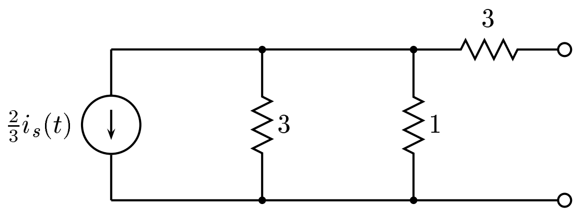
Solution: This problem can be worked many ways. This approach uses source substitutions, and the rules for replacing series and parallel connections of resistors by their equivalents to reduce the circuit to its Thévenin equivalent from which we can infer the $v - i$ relation. First we replace the current source in parallel with the 2Ω resistor by a voltage source in series with a resistor.



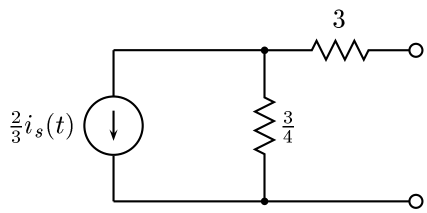
Now, by replacing the left part of the circuit by its equivalent, we get



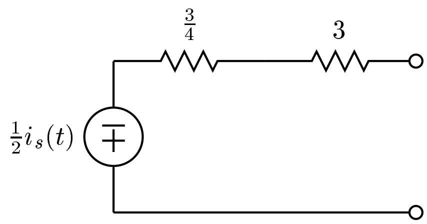
Now we can do another source substitution.



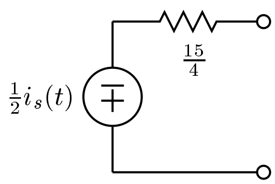
The next step is to combine the two resistors that are connected in parallel



Now we perform another source substitution



Finally, combining the series resistors gives us the Thévenin equivalent.



From the circuit we see that

$$v(t) = \frac{15}{4}i(t) - \frac{1}{2}i_s(t).$$
