GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

February 18, 2000

Problem Set #5–Solutions

Problem 5.1: A network has the $v - i$ characteristic shown graphically in Figure 1.

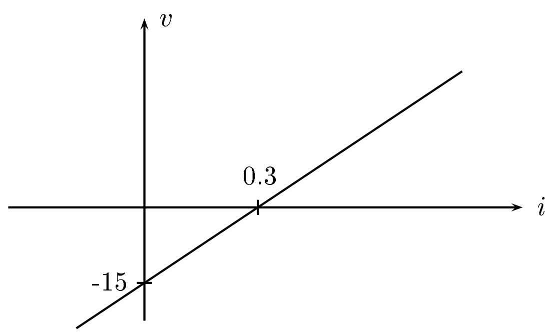


Figure 1: $v - i$ characteristic for Problem 5.1.

(a) Determine the Thévenin equivalent model corresponding to the network.

(b) Determine the Norton equivalent model corresponding to the network.

Solution:

(a) The $v - i$ characteristic of a Thévenin equivalent network is of the form

$$v = R_T i + v_T.$$

Thus, R_T is the slope of the line and $v = v_T$ when $i = 0$. For this graph the slope is

$$R_T = \frac{15}{0.3} = 50$$

and

$$v_T = -15$$

Therefore, the Thévenin equivalent network is that shown in Figure 2a.

(b) $i_N = -i_{sc}$, where i_{sc} is the value of i when $v = 0$. Appealing to the graph, $i_{sc} = 0.3$ and therefore, $i_N = -0.3$. This gives the Norton equivalent circuit shown in Figure 2b, where we have accomodated the minus sign by reversing the normal direction of the current source.

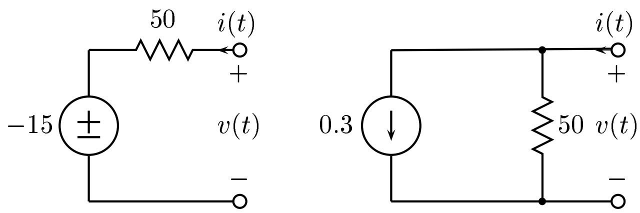


Figure 2: Equivalent circuits corresponding to the $v - i$ characteristic shown in Figure 1. (a) Thévenin equivalent circuit. (b) Norton equivalent circuit.

Problem 5.2: Consider the two two-terminal networks N_1 and N_2 shown in Figure 3.

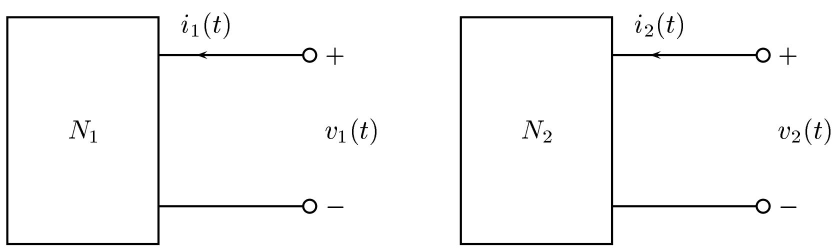


Figure 3: Networks for Problem 5.2

The two networks have the $v - i$ relations

$$N_1: \quad v_1(t) = 4i_1(t) - 8$$
$$N_2: \quad v_2(t) = 2i_2(t) + 3.$$

Determine the equilibrium values of $v(t)$ and $i(t)$ if the two networks are connected as shown in Figure 4. *Suggested approach:* Replace each network by its Thévenin equivalent network and then solve the resulting circuit.

Solution: Following the suggestion, we replace each network by its Thévenin equivalent. The two Thévenin equivalent circuits are shown in Figure 5. Now if we insert the Thévenin equivalent networks into Figure 4, we get the circuit shown in Figure 6.

Let the potential at the upper node where the three resistors are joined be denoted by $v(t)$. (This effectively locates the ground at the lower node.) Then, if we write a KCL equation at that (upper) node, we get

$$\frac{1}{5}[v(t) + 8] + \frac{1}{3}[v(t) - 3] + v(t) = 0$$

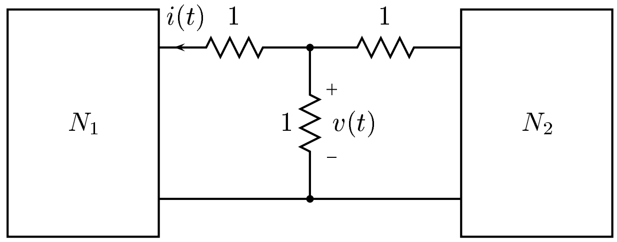


Figure 4: A circuit constructed from the two networks in Figure 3.

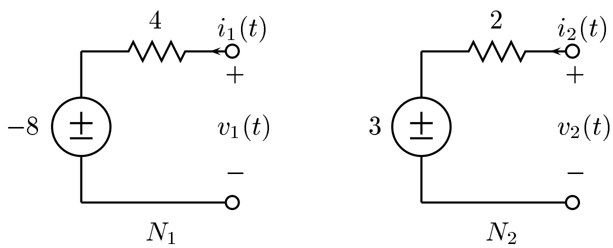


Figure 5: Thévenin equivalent networks for N_1 and N_2 in Problem 5.2.

From this

$$\frac{23}{15}v(t) = -\frac{3}{5}$$

or

$$v(t) = -\frac{9}{23}\text{V (constant).}$$

Problem 5.3: The two-terminal network N_1 in Figure 7 has the $v - i$ relation

$$v_1(t) = 2i_1(t) - 1$$

(a) Sketch a two-terminal network that is the Norton equivalent of N_1.

(b) Determine $v(t)$ as a function of $i(t)$.

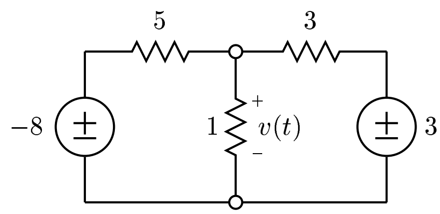


Figure 6: Circuit of Figure 4 with the Thévenin equivalent circuits included and the series resistors combined.

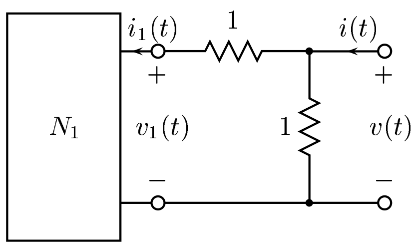
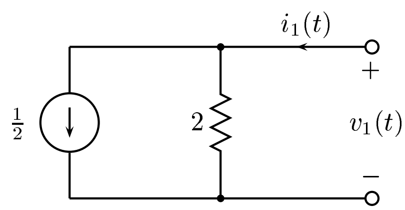


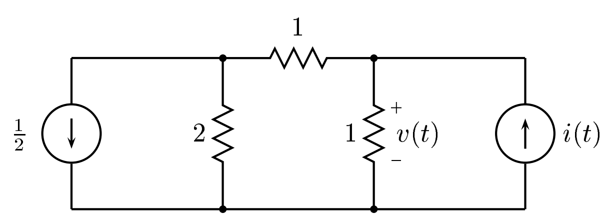
Figure 7: Circuit for Problem 5.3.

Solution:

(a)



(b) First, we can insert the Norton equivalent in place of the network N and attach a current source with the current $i(t)$. It is then sufficient to solve for the voltage drop $v(t)$ across the terminals, which is also the voltage drop across the 1Ω resistor.



It is straightforward to compute $v(t)$ by superposition

$$v(t) = \frac{3}{4}i(t) - \frac{1}{4}$$

Problem 5.4: A two-terminal network N has the $v - i$ characteristic

$$v_N(t) = 5i_N(t) - 3,$$

where $v_N(t)$ is the voltage drop across the terminals of the network and $i_N(t)$ is the current entering its + terminal. Determine the $v - i$ characteristics of the four two-terminal networks that are constructed from N and shown in Figure 8.

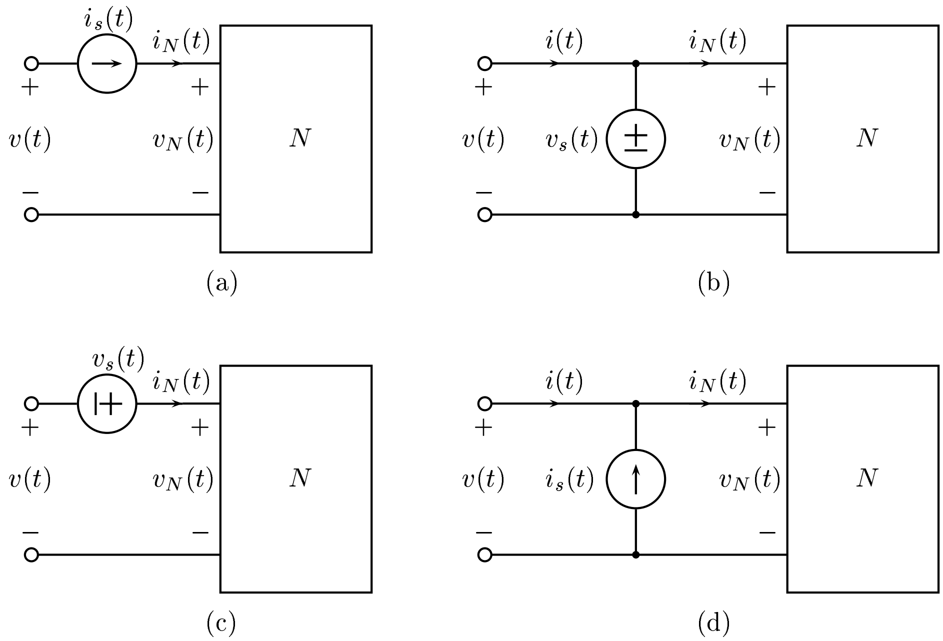


Figure 8: Four circuits constructed from the network N of Problem 5.4 whose $v - i$ relation needs to be determined.

Solution: For all four circuits, let i(t) be the current entering the terminals and let $v(t)$ be the voltage drop across the terminals.

(a) In this configuration $i(t) = i_s(t) = i_N(t)$ and $v(t)$ is unconstrained, i.e., this circuit is equivalent to the current source alone. The voltage drop across the terminals of N will be $v_N(t) = 5i_s(t) - 3$, but the voltage drop across the current source is unconstrained. This means that the voltage drop across the series connection of the current source and the network is also unconstrained.

(b) In this configuration $v(t) = v_s(t)$ and $i(t)$ is unconstrained, i.e., this circuit is equivalent to the voltage source alone. The reasoning is similar to that used in part (a) above.

(c) Here $i_N(t) = i(t)$ and $v(t) = v_N(t) - v_s(t)$. Therefore, incorporating the $v - i$ characteristic for N, we get

$$v_N(t) = v(t) + v_s(t) = 5i(t) - 3$$

or

$$v(t) = 5i(t) - [3 + v_s(t)].$$

(d) For this configuration $v_N(t) = v(t)$ and $i_N(t) = i(t) + i_s(t)$. Therefore,

$$v(t) = 5[i(t) + 5i_s(t)] - 3$$

or

$$v(t) = 5i(t) + [5i_s(t) - 3].$$

Problem 5.5: For the circuit in Figure 9 do the following:

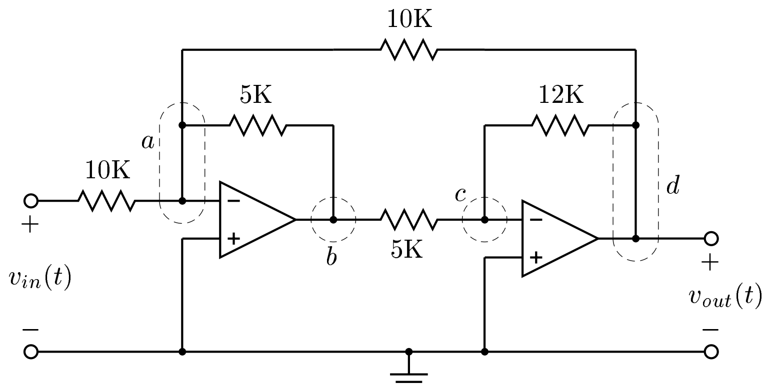


Figure 9: Circuit for Problem 5.5.

(a) Indicate the electric potential at each of the four indicated nodes. For each potential that is unknown prior to analyzing the circuit, give it a symbolic name, such as $e_a(t)$, etc.

(b) Identify the nodes at which you can write a meaningful KCL equation.

(c) Determine $v_{out}(t)$ in terms of $v_{in}(t)$.

Solution:

(a) node a**:** $e_a(t) = 0$
node b**:** $e_b(t)$, initially unknown
node c**:** $e_c(t) = 0$
node d**:** $e_d(t) = v_{out}(t)$

(b) We can write KCL equations at nodes a and c. (Nodes b and d are output nodes of opamps.)

(c)

$$\text{node } a: \quad -\frac{v_{in}(t)}{10K} - \frac{e_b(t)}{5K} - \frac{v_{out}(t)}{10K} = 0$$
$$\text{node } c: \quad -\frac{e_b(t)}{5K} - \frac{v_{out}(t)}{12K} = 0$$

From the second equation

$$\frac{e_b(t)}{5K} = -\frac{v_{out}(t)}{12K}$$

Substituting into the first equation

$$-\frac{v_{in}(t)}{10K} + \frac{v_{out}(t)}{12K} - \frac{v_{out}(t)}{10K} = 0$$

from which

$$v_{out}(t) = -6v_{in}(t).$$

Problem 5.6: Express the output voltage $v_{out}(t)$ in terms of the input voltage $v_{in}(t)$ for the circuit in Figure 10.

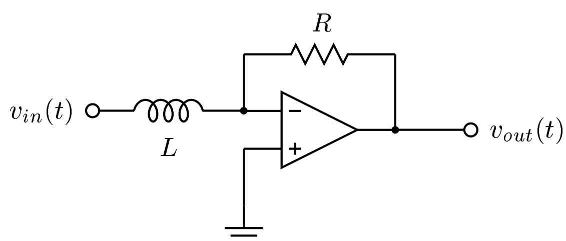


Figure 10: Circuit for Problem 5.6.

Solution: Let the current entering the node connected to the inverting input through the inductor be $i_\ell(t)$. Then the KCL equation at that node is

$$i_\ell(t) + \frac{1}{R}v_{out}(t) = 0$$

Substituting the $v - i$ relation for the inductor (in integral form gives

$$\frac{1}{L}\int_{t_0}^{t} v_{in}(\beta)\,d\beta + i_\ell(t_0) + \frac{1}{R}v_{out}(t) = 0,$$

which we can solve for $v_{out}(t)$.

$$v_{out}(t) = -\frac{R}{L}\int_{t_0}^{t} v_{in}(\beta)\,d\beta - Ri_\ell(t_0).$$
