GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040

Circuit Analysis

March 17, 2000

Problem Set #8–Solutions

Problem 8.1: For the circuit shown in Figure 1.

(a) Find $v_r(t)$ for $t > 0$ if $i_\ell(0) = 0$.

(b) Find $v_r(t)$ for $t > 0$ if $i_\ell(0) = 5$.

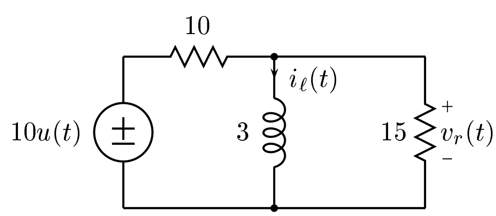


Figure 1: Circuit for Problem 8.1.

Solution:

(a) At $t = 0$ the inductor looks like an open circuit.

$$v_r(0) = \frac{15}{25} \cdot 10 = 6$$

At $t = \infty$ the inductor looks like a short circuit.

$$v_r(\infty) = 0.$$

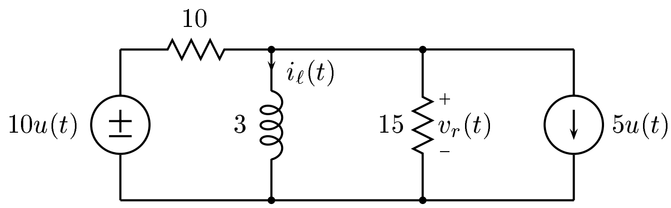
We turn off the source to determine the time constant

$$\tau = \frac{L}{R_{eq}} = \frac{3}{6} = \frac{1}{2}.$$

Therefore,

$$v_r(t) = 6e^{-2t} \quad t \geq 0.$$

(b) We can incorporate a current source to handle the initial condition. This changes the circuit to



The time constant is the same as before. The value at $t = \infty$ is also the same as before. At $t = 0$

$$v_r(0) = 6 - 30 = -24.$$

Therefore,

$$v_r(t) = -24e^{-2t} \quad t \geq 0.$$

Problem 8.2: Use the inspection technique to determine the current $i(t)$ in the circuit in Figure 2.

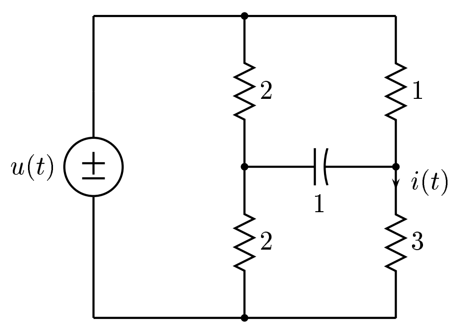


Figure 2: Circuit to be solved by inspection in Problem 8.2.

Solution: At $t = 0$ the capacitor looks like a short circuit. In this case the upper resistors are in parallel and have an equivalent resistance of $2/3\Omega$. The two lower resistors are also in parallel with an effective resistance of $6/5\Omega$. Therefore, the voltage across the 3Ω resistor is (by a voltage divider)

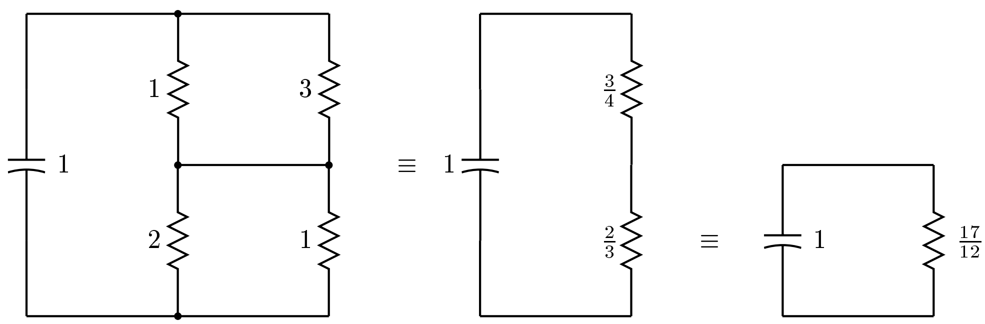
$$\frac{\frac{6}{5}}{\frac{6}{5} + \frac{2}{3}} = \frac{9}{14} \text{ volts.}$$

and $i(0) = \frac{9}{14}\frac{1}{3} = \frac{3}{14}$ volts.

At $t = \infty$ the capacitor looks like an open circuit. By a voltage divider, the voltage across the 3Ω resistor is $3/4$ volt and the current is $1/4$ amp.

$$i(\infty) = \frac{1}{4} \text{ amp}$$

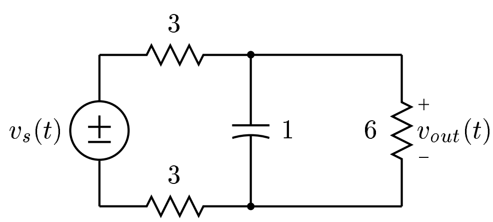
When the voltage source is turned off the circuit reduces to



Therefore, $\tau = RC = \frac{17}{12}$ and

$$i(t) = \frac{1}{4} + \left(\frac{3}{14} - \frac{1}{4}\right) e^{-\frac{12}{17}t} = \frac{1}{4} - \frac{1}{28} e^{-\frac{12}{17}t}$$

Problem 8.3: The circuit in Figure 3 is at initial rest and $v_s(t) = u(t)$.



(a) Determine $v_{out}(0)$.

(b) Determine $v_{out}(\infty)$.

(c) Determine $v_{out}(t)$ for all t.

Figure 3: Circuit for Problem 8.3.

Solution:

(a) At $t = 0$ the capacitor looks like a short circuit. Therefore,

$$v_{out}(0) = 0.$$

(b) At $t = \infty$ the capacitor looks like an open circuit. Therefore, by a voltge divider

$$v_{out}(\infty) = \frac{1}{2}.$$

(c) The time constant $\tau = R_{eq}C = 3$, since when the voltage source is turned off, the capacitor is connected in parallel with two 6Ω resistors. Therefore,

$$v_{out}(t) = \frac{1}{2}\left(1 - e^{-t/3}\right) u(t).$$

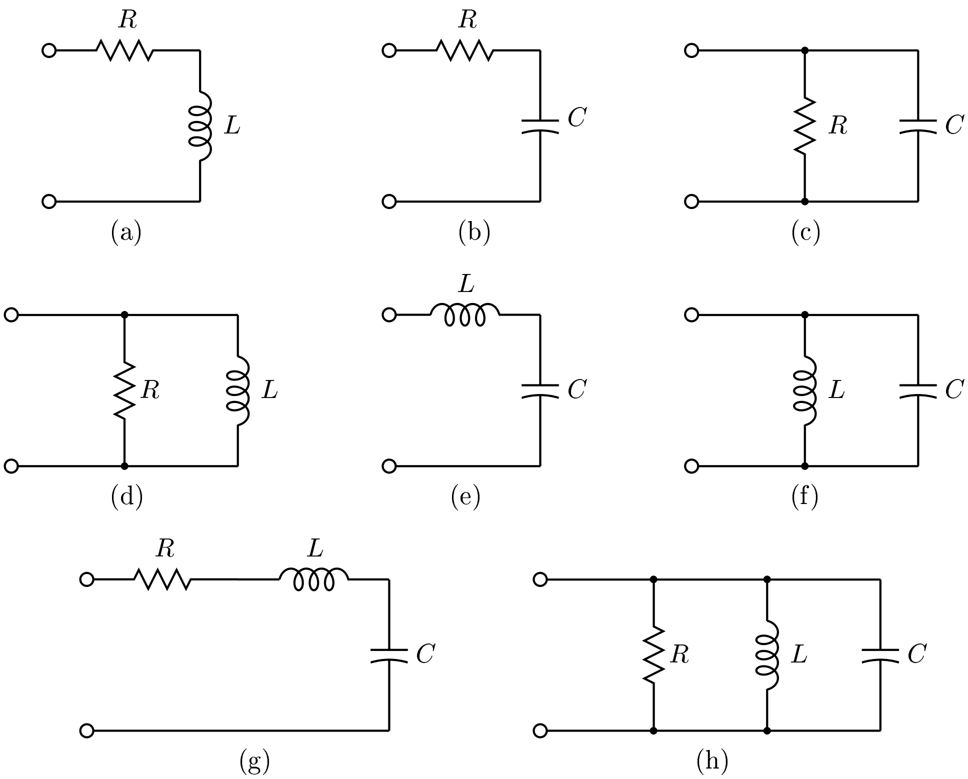


Figure 4: Circuits for Problem 6.9.

Problem 8.4: For each of the networks in Figure 4, determine the equivalent impedance. Express your answers as ratios of polynomials in s.

Solution:

(a) $Z_a(s) = R + Ls$

(b) $Z_b(s) = R + \frac{1}{CS} = \frac{RCs+1}{Cs}$

(c) $Z_c(s) = \frac{\frac{R}{Cs}}{R+\frac{1}{Cs}} = \frac{R}{RCs+1}$

(d) $Z_d(s) = \frac{RLs}{R+Ls}$

(e) $Z_e(s) = Ls + \frac{1}{Cs} = \frac{LCs^2+1}{Cs}$

(f) $Z_f(s) = \frac{\frac{L}{C}}{Ls+\frac{1}{Cs}} = \frac{LCs^2+1}{Cs}$

(g) $Z_g(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs}$

(h) $Z_h(s) = \frac{1}{\frac{1}{R}+\frac{1}{Ls}+Cs} = \frac{RLs}{RLCs^2+Ls+R}$

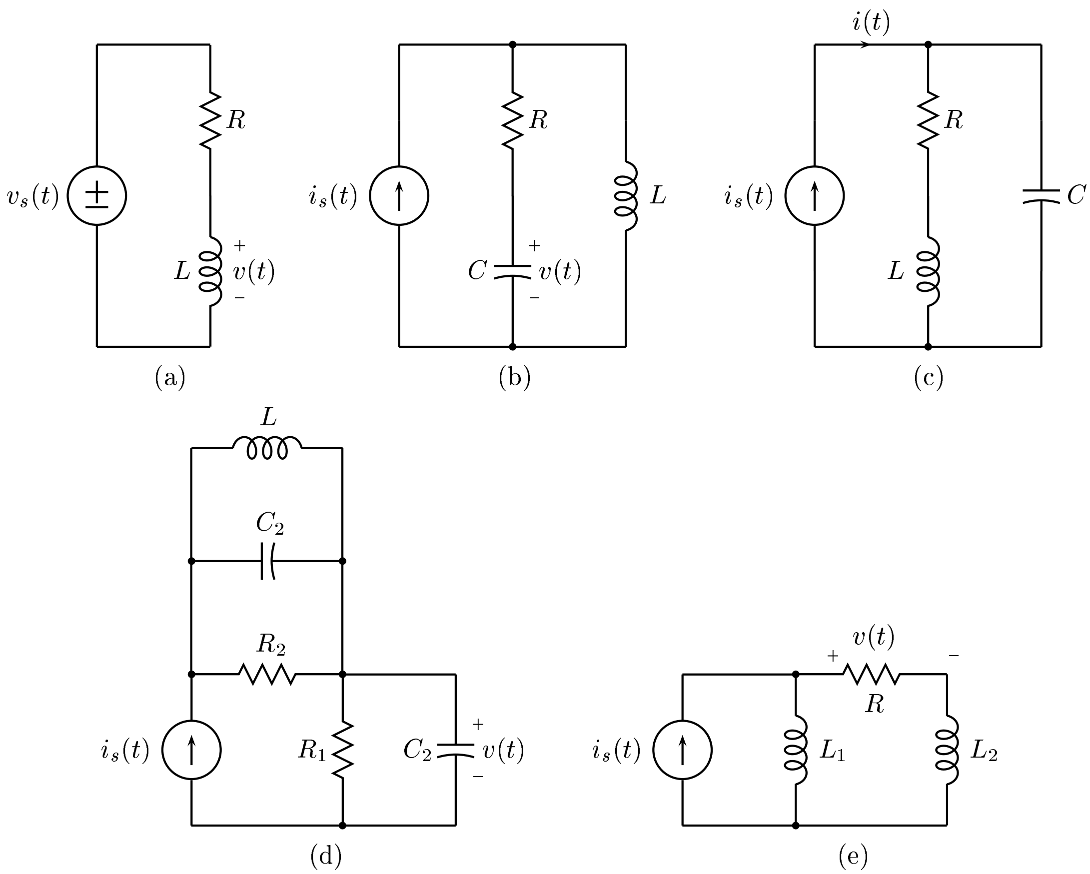


Figure 5: Networks for Problem 8.5.

Problem 8.5: For each of the networks in Figure 5 determine the Laplace domain formula that relates the indicated output variable to the source variable. Your answers should be expressed in terms of the Laplace transforms of the source variables and the Laplace domain variable s.

Solution:

(a) $H_a(s) = \frac{Ls}{Ls+r}$

(b) $H_b(s) = \frac{1}{Cs} \cdot \frac{Ls}{R+\frac{1}{Cs}+Ls} = \frac{Ls}{LCs^2+RCs+1}$

(c) $H_c(s) = 1$

(d) $H_d(s) = \frac{1}{C_1 s} \cdot \frac{R_1}{R_1+\frac{1}{C_1 s}} = \frac{R_1}{R_1 C_1 s+1}$

(e) $H_e(s) = \frac{L_1 s}{R+(L_1+L_2)s} \cdot R = \frac{RL_1 s}{(L_1+L_2)s+R}$