

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course ECE 2040
Circuit Analysis

March 24, 2000

Problem Set #9–Solutions

Problem 9.1: Find $v_{out}(t)$ for $t > 0$ when $v_{in}(t) = \cos(1000t)$ and $v_{out}(0) = 0$ for the circuit in Figure 1.

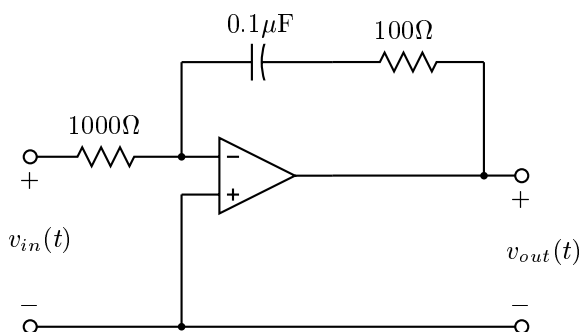


Figure 1: Circuit for Problem 9.1.

Solution: This circuit looks like an inverting amplifier with

$$Z_i(s) = 1000$$
$$Z_f(s) = \frac{1}{10^{-5}s} + 100 = \frac{10^5}{s} + 100.$$

Therefore,

$$V_{out}(s) = -\frac{Z_f(s)}{Z_i(s)} \cdot v_{in}(s)$$
$$= -\frac{\frac{10^5}{s} + 100}{1000} V_{in}(s) = \left(\frac{100}{s} + 0.1 \right) v_{in}(s).$$

Since

$$V_{in}(s) = \frac{s}{s^2 + 10^6}$$

we have

$$V_{out}(s) = \frac{100}{s^2 + 10^6} + \frac{0.1s}{s^2 + 10^6}.$$

From the table of Laplace transforms and the linearity property

$$v_{out}(t) = 0.1 \sin 1000t + 0.1 \cos 1000t, \quad t > 0.$$

Problem 9.2: (a) For the circuit in Figure 2 determine the system function $H(s)$ that relates the output $i(t)$ to the input $i_s(t)$, i.e. find

$$H(s) = \frac{I(s)}{I_s(s)}$$

(b) Determine $i(t)$ for all values of t if $i_s(t) = u(t)$. Assume that the circuit is at initial rest for $t < 0$.

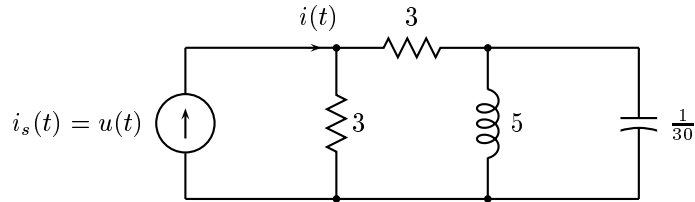


Figure 2: Circuit for Problem 9.2.

Solution:

(a)

$$H(s) = \frac{3}{6 + \frac{5s \cdot \frac{30}{s}}{5s + \frac{30}{s}}} \cdot \frac{\frac{30}{s}}{5s + \frac{30}{s}} = \frac{3 \cdot \frac{30}{s}}{30s + \frac{180}{s} + 150} = \frac{3}{s^2 + 5s + 6}$$

(b)

$$I(s) = \frac{1}{s} \cdot \frac{3}{s^2 + 5s + 6} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = \lim_{s \rightarrow 0} \frac{3}{(s+3)(s+2)} = \frac{1}{2}$$

$$B = \lim_{s \rightarrow -2} \frac{3}{s(s+3)} = -\frac{3}{2}$$

$$C = \lim_{s \rightarrow -3} \frac{3}{s(s+2)} = 1$$

Therefore,

$$i(t) = \left(\frac{1}{2} - \frac{3}{2}e^{-2t} + e^{-3t} \right) u(t)$$

Problem 9.3: Find the system function $H(s) = V_{out}(s)/V_{in}(s)$ for the circuit in Figure 3.

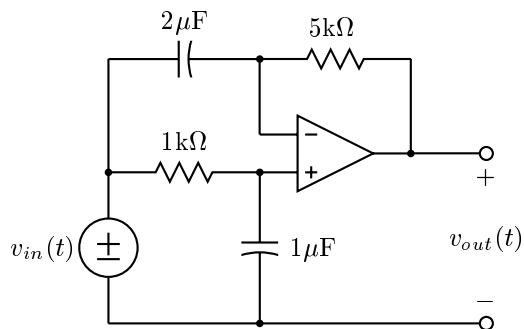


Figure 3: Circuit for Problem 9.3.

Solution: Let the node potentials at the two nodes connected to the inputs of the opamp be $E(s)$. (They are the same.) Then we get for KCL equations

$$C_1 s[E(s) - V_{in}(s)] + \frac{1}{R_2}[E(s) - V_{out}(s)] = 0$$

$$\frac{1}{R_1}[E(s) - V_{in}(s)] + C_2 s E(s) = 0$$

From the second equation

$$E(s) = \frac{1}{(1 + R_1 C_2 s)} V_{in}(s).$$

From the first

$$[1 + C_1 R_2 s] E(s) - R_2 C_1 s V_{in}(s) = V_{out}(s)$$

$$\left[\frac{1 + C_1 R_2 s}{1 + C_2 R_1 s} - R_2 C_1 s \right] V_{in}(s) = V_{out}(s)$$

From this

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1 - C_1 C_2 R_1 R_2 s^2}{1 + C_2 R_1 s} = \frac{1 - 10^{-5} s^2}{1 + 10^{-3} s}$$

Problem 9.4: The network of Figure 4 is initially at rest. Determine the voltage $v(t)$ for each of the inputs below:

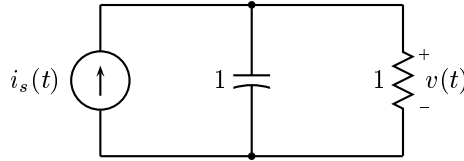


Figure 4: Circuit for Problem 9.4.

- (a) $i_s(t) = u(t)$
- (b) $i_s(t) = (\sin t)u(t)$
- (c) $i_s(t) = tu(t)$

Solution: For this circuit

$$H(s) = \frac{V(s)}{I_s(s)} = Z_{cq}(s) = \frac{1}{s+1}.$$

Since the circuit is at initial rest, all three outputs will be zero for $t < 0$.

- (a) If $i_s(t) = u(t)$, then $I_s(s) = \frac{1}{s}$ and

$$V_a(s) = H(s)I_s(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}.$$

Therefore,

$$v_a(t) = (1 - e^{-t})u(t).$$

- (b) If $i_s(t) = (\sin t)u(t)$, then $I_s(s) = \frac{1}{s^2+1}$ and

$$V_b(s) = H(s)I_s(s) = \frac{1}{(s+1)(s^2+1)} = \frac{\frac{1}{2}}{s+1} - \frac{\frac{1}{2}s}{s^2+1} + \frac{\frac{1}{2}}{s^2+1}.$$

Therefore,

$$v_b(t) = \frac{1}{2} (1 - \cos t + \sin t) u(t)$$

- (c) If $i_s(t) = tu(t)$, then $I_s(s) = \frac{1}{s^2}$ and

$$V_c(s) = \frac{1}{s^2(s+1)} = \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}.$$

Therefore,

$$v_c(t) = [t - 1 + e^{-t}] u(t)$$

Problem 9.5: (a) For the circuit in Figure 5 determine the system function that relates $V_c(s)$ and $V_s(s)$.

- (b) Find $v_c(t)$ for all time if the circuit is at initial rest and $v_s(t) = u(t)$.

- (c) Find $v_c(t)$ for all time if the circuit is at initial rest and $v_s(t) = 2\delta(t)$.

Solution:

- (a) The system function is

$$H(s) = \frac{V_c(s)}{V_s(s)} = \frac{\frac{4}{s}}{s + 4 + \frac{4}{s}} = \frac{4}{s^2 + 4s + 4} = \frac{4}{(s + 2)^2}$$

- (b) Since $V_c(s) = H(s)V_s(s)$, we have

$$V_c(s) = \frac{4}{(s + 2)^2} \cdot \frac{1}{s} = \frac{1}{s} - \frac{1}{s + 2} - \frac{2}{(s + 2)^2}$$

Computing the appropriate inverse Laplace transforms

$$v(t) = [1 - (2t + 1)e^{-2t}]u(t)$$

- (c) Since $V_c(s) = \frac{8}{(s+2)^2}$, $v(t) = 8te^{-2t}u(t)$.
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Problem 9.6: A parallel RC circuit is connected in series with an unknown circuit N excited by a current source as shown in Figure 6(a). The current source waveform is shown in Figure 6(b). The goal of the problem is to design the network N so that the voltage $v(t)$ will be that shown in Figure 6(c).

- (a) Use Laplace transform methods to determine the voltage $v_2(t)$.
 (b) Sketch $v_1(t)$ for $t \geq 0$.
 (c) Determine the impedance $Z_1(s)$ of the circuit N .
 (d) Determine the circuit N .

Solution:

- (a) Notice that the whole current $i_s(t)$ passes through the parallel connection. Therefore, we can find $v_2(t)$ by finding the voltage drop in the following circuit.

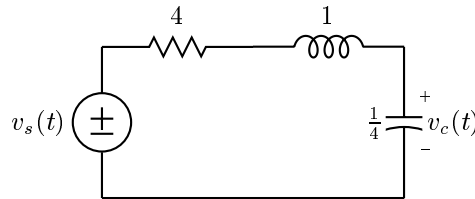


Figure 5: Circuit for Problem 9.5.

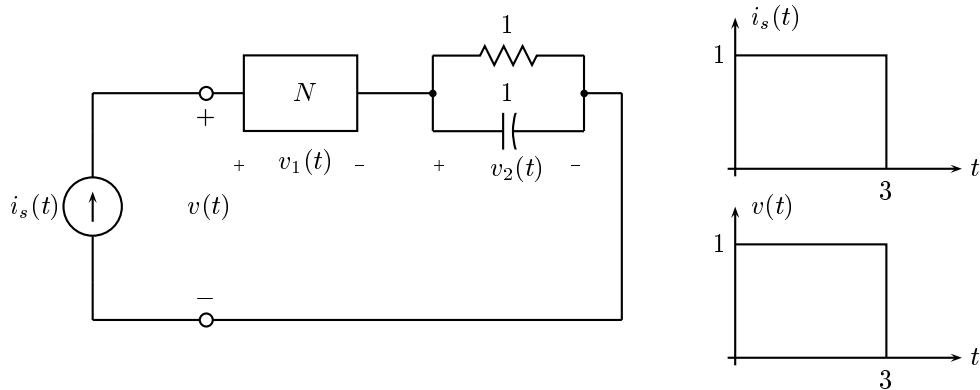
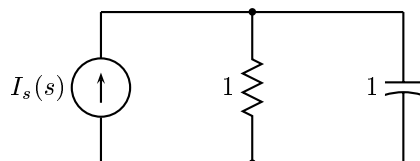


Figure 6: Circuit for Problem 9.6 with the current excitation and voltage response shown. The voltage $v_1(t)$ and the circuit N are to be determined.



Let the voltage drop across the capacitor be $s(t)$, when $i_s(t) = u(t)$. Then

$$S(s) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} \cdot \frac{1}{s} = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

and

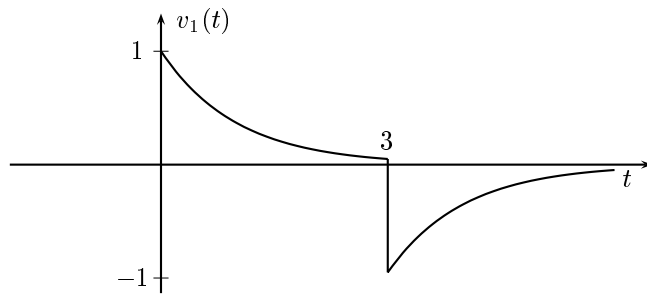
$$s(t) = [1 - e^{-t}]u(t).$$

Therefore,

$$\begin{aligned} v_2(t) &= s(t) - s(t-3) = [1 - e^{-t}]u(t) - [1 - e^{-(t-3)}]u(t-3) \\ &= \begin{cases} 0, & t < 0 \\ 1 - e^{-t}, & 0 \leq t \leq 3 \\ (e^3 - 1)e^{-t}, & 3 < t \end{cases} \end{aligned}$$

(b) $v_1(t) = v(t) - v_2(t)$. Therefore

$$v_1(t) = \begin{cases} 0, & t < 0 \\ e^{-t}, & 0 < t < 3 \\ -(e^3 - 1)e^{-t}, & 3 < t \end{cases}$$



- (c) The impedance of the overall system should have a constant value of 1. Therefore, if $Z_2(s)$ denotes the impedance of the parallel RL

$$Z_1(s) + Z_2(s) = 1.$$

This means that

$$Z_1(s) = 1 - Z_2(s) = 1 - \frac{1}{s+1} = \frac{s}{s+1}.$$

- (d) One circuit that will have this impedance is a parallel RL .

