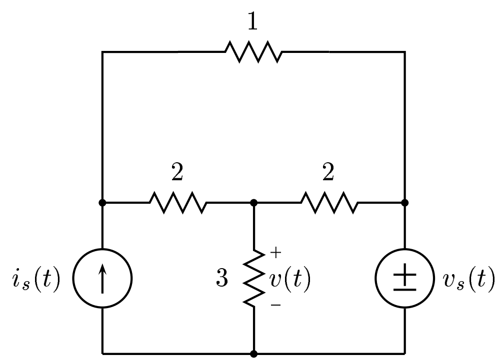
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
**Circuit Analysis**

Quiz #2–Solutions

Wednesday, February 25, 2000

**Problem Q2.1:**
Solve for the voltage $v(t)$ using the mesh method.

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**Solution:** Let the mesh current (clockwise) in the upper mesh be $i_\alpha(t)$ and in the lower right mesh, let it be $i_\beta(t)$. Assign a mesh current $i_s(t)$ to the lower left mesh. Then

$$\begin{aligned} \alpha \text{ mesh:} \quad & i_\alpha(t) + 2[i_\alpha(t) - i_\beta(t)] + 2[i_\alpha(t) - i_s(t)] = 0 \\ \beta \text{ mesh:} \quad & 3[i_\beta(t) - i_s(t)] + 2[i_\beta(t) - i_\alpha(t)] + v_s(t) = 0 \end{aligned}$$

Rearranging

$$\begin{aligned} 5i_\alpha(t) - 2\imath_\beta(t) &= 2i_s(t) \\ -2i_\alpha(t) + 5\imath_\beta(t) &= -v_s(t) + 3i_s(t) \end{aligned}$$

Multiply the first equation by 2, the lower one by 5 and add.

$$21i_\beta(t) = 19i_s(t) - 5v_s(t)$$

or

$$i_\beta(t) = \frac{19}{21}i_s(t) - \frac{5}{21}v_s(t).$$

But

$$v(t) = 3[i_s(t) - \imath_\beta(t)] = 3\left[\frac{2}{21}i_s(t) + \frac{5}{21}v_s(t)\right]$$

or

$$v(t) = \frac{2}{7} i_s(t) + \frac{5}{7} v_s(t).$$

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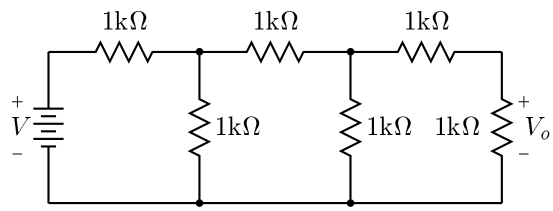
**Problem Q2.2:**

For the circuit below the source voltage on the left is a battery (i.e., a constant voltage source).




Determine the value of $V$ required to produce a value for $V_o$ of 1 volt.

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**Solution:** Number the six resistors $R_1$, $R_2$, etc. from left to right. To get 1 volt across $R_6$ requires a current flowing through it of 1 mA. Since this current also flows through $R_5$, there will be a 2 volt drop across the series combination of $R_5$ and $R_6$. This means that there will be a 2 volt drop across $R_4$ and a 2 mA current flowing through it. Working backwards, this means that there will be a 3 mA current through $R_3$ and a voltage drop of 3 volts. A 3 volt drop across $R_3$ with a 2 volt drop across $R_4$ means a 5 volt drop across $R_2$ and a current of 5 mA. Finally there will be an 8 mA current through $R_1$ and an 8 volt drop. Therefore, $V = 13$ volts. (Notice that the sequence of voltages [and the sequence of currents]forms a Fibonacci series.)

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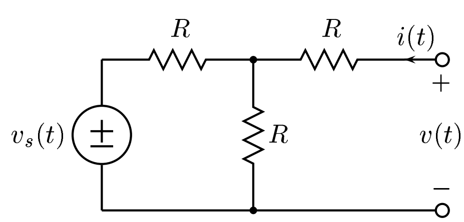
**Problem Q2.3:**



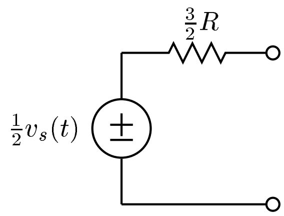

(a) Find the Thévenin equivalent network corresponding to the above two-terminal circuit.

(b) Find the Norton equivalent network.

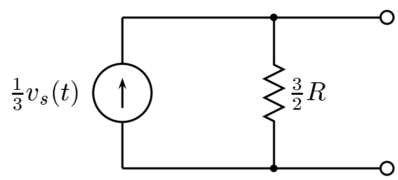
**Solution:**

$$v_{oc}(t) = \frac{1}{2} v_s(t)$$
$$R_{eq} = \frac{R}{2} + R = \frac{3}{2} R$$

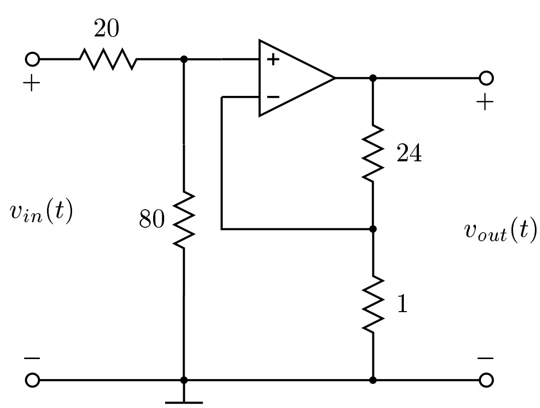
(a)




(b)




**Problem Q2.4:**
Determine $v_{out}(t)$ in terms of $v_{in}(t)$.




**Solution:** Circuits containing opamps are normally analyzed using the node method, appropriately modified. For this circuit we need to write KCL equations at the two nodes connected to the two inputs of the opamp. The potential at each of these is the same; call that potential $e(t)$.

$$\begin{aligned} + \text{ node:} \quad & \frac{1}{20}[e(t) - v_{in}(t)] + \frac{1}{80}e(t) = 0 \\ - \text{ node:} \quad & \frac{1}{24}[e(t) - v_{out}(t)] + e(t) = 0 \end{aligned}$$

From the first equation we find

$$e(t) = \frac{4}{5}v_{in}(t).$$

Substituting this value into the second equation gives

$$\begin{aligned} & \frac{1}{24}[\frac{4}{5}v_{in}(t) - v_{out}(t)] + \frac{4}{5}v_{in}(t) = 0 \\ & \frac{5}{6}v_{in}(t) - \frac{1}{24}v_{out}(t) = 0 \\ & v_{out}(t) = 20v_{in}(t). \end{aligned}$$

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