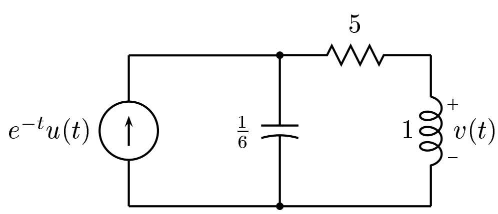
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #3–Solutions

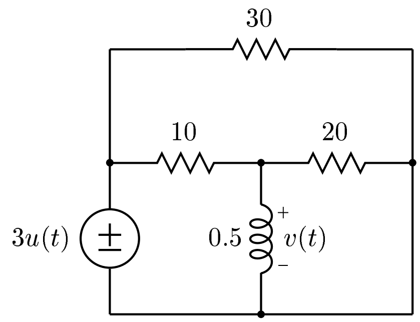
Monday, March 29, 2000

Problem Q3.1: In the circuit below solve for $v(t)$ for all t.



Solution: Clearly
$$V(s) = sI_\ell(s)$$
and we can determine $I_\ell(s)$ by using a current divider. Therefore,
$$V(s) = s\left(\frac{\frac{6}{s}}{s+5+\frac{6}{s}}\right)\left(\frac{1}{s+1}\right) = \frac{6s}{(s+1)(s+2)(s+3)}.$$
Performing a partial fraction expansion gives
$$V(s) = \frac{-3}{s+1} + \frac{12}{s+2} + \frac{-9}{s+3}$$
and
$$v(t) = \left(-3e^{-t} + 12e^{-2t} - 9e^{-3t}\right)u(t).$$
The response is zero for $t < 0$ because the input is zero in that range.

Problem Q3.2: Consider the first-order circuit below



(a) Find $v(0)$.

(b) Find $v(\infty)$.

(c) Find $v(t)$ for $t > 0$.

Solution:

(a) At $t = 0$ the inductor looks like an open circuit. The voltage drop across the inductor is the same as the voltage drop across the 20 Ohm resistor. Using a voltage divider, this is seen to be 2 Volts.

$$v(0) = 2V$$

(b) At $t = \infty$, the inductor becomes a short circuit. The voltage drop across it is then zero.

$$v(\infty) = 0.$$

(c) To get the Thevenin resistance, we turn off the voltage source, replacing it by a short circuit. The inductor is then seen to be connected in parallel with the 10 Ohm resistor and also with the 20 Ohm resistor. (The 30 Ohm resistor is shorted out.) Therefore

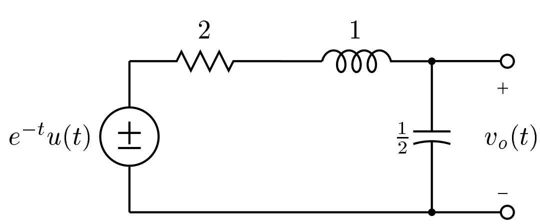
$$R_T = \frac{20}{3}$$

$$\tau = \frac{L}{R_T} = \frac{3}{40}$$

and

$$v(t) = 2e^{-40t/3} \quad t > 0.$$

Problem Q3.3: Find the Thevenin equivalent network corresponding to the one-port circuit below.



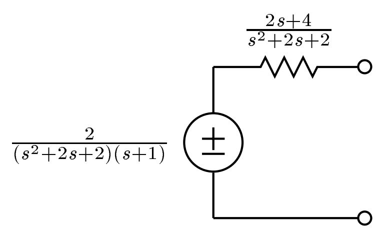
Solution: To find the Thevenin equivalent network in the Laplace domain, we simply need to find the equivalent impedance with the voltage source turned off and the open-circuit voltage.

$$Z_T(s) = \frac{\frac{2}{s} \cdot (s+2)}{2+2+\frac{2}{s}} = \frac{2s+4}{s^2+2s+2}$$

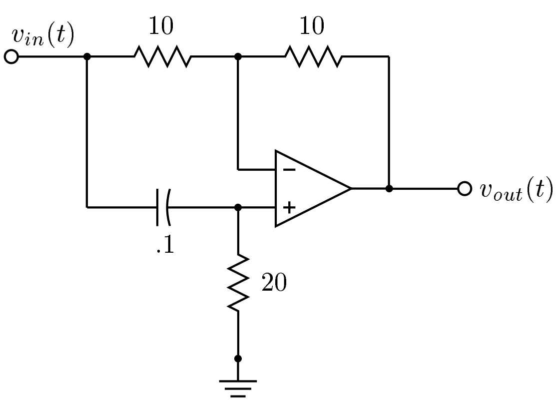
To get the open circuit voltage, we can apply a voltage divider

$$\begin{aligned} V_{oc}(s) &= \frac{\frac{2}{s}}{s+2+\frac{2}{s}} \cdot \frac{1}{s+1} \\ &= \frac{2}{(s^2+2s+2)(s+1)} \end{aligned}$$

This gives the equivalent network (Laplace domain)



Problem Q3.4: This problem asks you to analyze the following circuit.



(a) Determine the system function of the circuit if $v_{in}(t)$ is the input signal and $v_{out}(t)$ is the output signal.

(b) Determine the impulse response of the circuit.

Solution:

(a) We are looking for

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

To solve for $V_{out}(s)$, we write a KCL equation at the nodes connected to the two inputs of the opamp. Let the potential at each of those nodes by $E(s)$. Then,

$$\begin{aligned} (.1)s[E(s) - V_{in}(s)] + \frac{1}{20}E(s) = 0 \\ \frac{1}{10}[E(s) - V_{in}(s)] + \frac{1}{10}[E(s) - V_{out}(s)] = 0 \end{aligned}$$

From the first equation

$$(2s+1)E(s) = 2sV_{in}(s)$$

or

$$E(s) = V_{in}(s)\frac{2s}{2s+1}.$$

From the second equation

$$2E(s) = V_{in}(s) + V_{out}(s).$$

Therefore,

$$\frac{4s}{2s+1}V_{in}(s) = V_{in}(s) + V_{out}(s)$$

and

$$\begin{aligned} V_{out}(s) &= \frac{2s-1}{2s+1}V_{in}(s) \\ &= \frac{s-\frac{1}{2}}{s+\frac{1}{2}}V_{in}(s) \end{aligned}$$

and

$$H(s) = \frac{s-\frac{1}{2}}{s+\frac{1}{2}}$$

(b) Performing a partial fraction expansion

$$H(s) = 1 - \frac{1}{s+\frac{1}{2}}.$$

Therefore,

$$h(t) = \delta(t) - e^{-t/2}u(t).$$