

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2040
Circuit Analysis

Quiz #1 – Solutions

Problem Q1.1: In the circuit shown in Figure 1 the current source waveform is

$$i_s(t) = 5 \cos(50t).$$

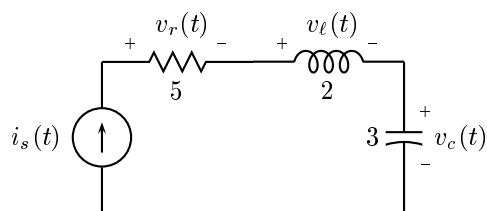


Figure 1:

- (a) (5 pts) Determine $v_r(t)$.
- (b) (10 pts) Determine $v_l(t)$.
- (c) (10 pts) Determine $v_c(t)$.

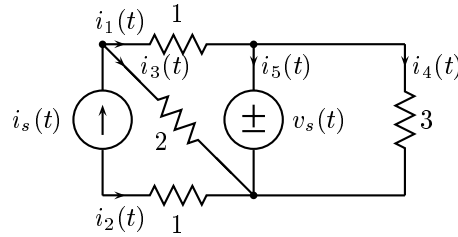
Solution:

(a) $v_r(t) = 5i_r(t) = 25 \cos(50t)$

(b) $v_l(t) = L \frac{dv_l(t)}{dt} = 2 \frac{d}{dt} (5 \cos(50t)) = -500 \sin(50t)$

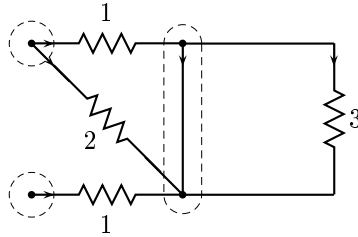
(c) $v_c(t) = \frac{1}{C} \int_{-\infty}^t i_c(\tau) d\tau = \frac{5}{3} \int_{-\infty}^t \cos(50\tau) d\tau = \frac{1}{30} \sin(50t)$.

Problem Q1.2:



- In the circuit above, what is the minimum number of KCL equations that you will need to write to determine the equilibrium solution?
- What is the minimum number of KVL equations that you will need to write to determine the equilibrium solution?
- Write the KCL equations using the current variables defined on the figure.
- Write the KVL equations in terms of the currents and $v_s(t)$ by incorporating the element relations.
- Write the complete set of KCL and KVL sufficient to determine $i_1(t)$, $i_2(t)$, $i_3(t)$, $i_4(t)$, and $i_5(t)$ in matrix-vector form.

Solution: The basic network corresponding to this circuit is shown below.



The currents are defined in the original circuit.

- The nodes of the basic network are circled in the above figure. There are three of them, which means that we will need to write two KCL equations.
- The basic network contains two meshes. Therefore, we will need to write two KVL equations.
- The three KCL equations are: (Note that any two of these are sufficient.)

$$\begin{aligned} i_1(t) + i_3(t) &= i_s(t) \\ i_2(t) &= -i_s(t) \\ i_1(t) + i_2(t) + i_3(t) &= 0 \end{aligned}$$

Notice that for the last equation, which corresponds to the supernode, $i_4(t)$ both enters and leaves the supernode, so that both currents cancel out.

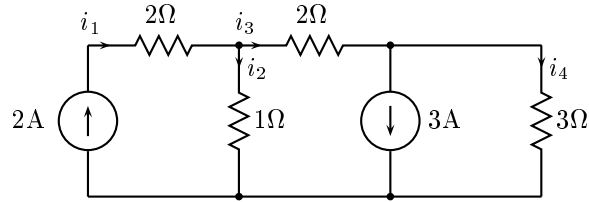
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$$\begin{aligned} i_1(t) - 2i_3(t) &= -v_s(t) \\ 3i_4(t) &= v_s(t) \end{aligned}$$

(e)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \\ i_4(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} i_s(t) + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} v_s(t).$$

Problem Q1.3: In the circuit below each of the current sources generates a constant current. Compute the values of the four resistor currents, i_1 , i_2 , i_3 , and i_4 .



Solution: First, notice that we can determine i_1 (and v_1) immediately, before determining any of the other element variables.

$$i_1 = 2A$$

We can write KCL equations at the nodes at the two ends of the rightmost 2Ω resistor. These are

$$i_2 + i_3 = 2 \implies i_2 = 2 - i_3$$

$$i_3 - i_4 = 3 \implies i_4 = i_3 - 3$$

From KVL around the single mesh of the basic network

$$-i_2(t) + 2i_3 + 3i_4 = 0$$

Substituting for i_2 and i_4 gives

$$-2 + i_3 + 2i_3 + 3i_3 - 9 = 0$$

from which

$$i_3 = \frac{11}{6}A$$

Finally,

$$i_2 = 2 - i_3 = \frac{1}{6}A$$

$$i_4 = i_3 - 3 = -\frac{7}{6}A.$$

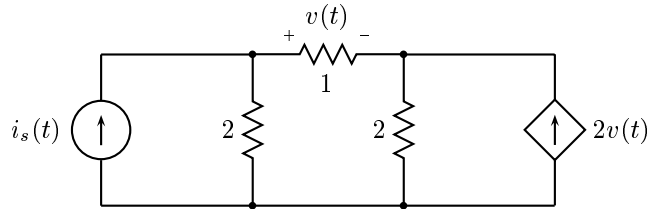


Figure 2:

Problem Q1.4: In the circuit below, which contains a voltage-dependent current source, determine $v(t)$, by any method you choose.

Solution: We could solve this problem by setting up and solving the usual set of KVL equations, KCL equations, and element relations. There is nothing wrong with approaching this problem that way. An alternative, however, is to evaluate some of the voltages and currents in the circuit whose value can be immediately determined and then use these values to evaluate others. That is the approach discussed here.

The current flowing left to right through the horizontal 1Ω resistor is $v(t)$. Since a current $2v(t)$ comes from the dependent current source, KCL tells us that a current of $3v(t)$ must flow through the rightmost 2Ω resistor from top to bottom. This will induce a voltage drop of $6v(t)$. This means that there is a voltage drop of $7v(t)$ across the leftmost 2Ω resistor (by KVL) and a current of $\frac{7}{2}v(t)$ flowing through it. Since the current through the horizontal resistor was $v(t)$, by KCL we have finally

$$v(t) + \frac{7}{2}v(t) = i_s(t)$$

or

$$v(t) = \frac{2}{9}i_s(t).$$
