

GEORGIA INSTITUTE OF TECHNOLOGY  
School of Electrical and Computer Engineering

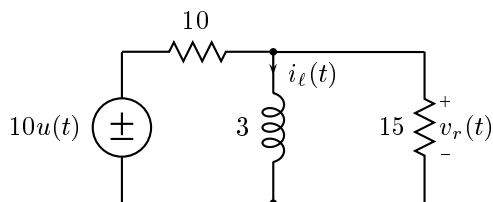
Course EE 2250  
Electric Circuit Analysis

May 13, 1999

**Problem Set #6–Solutions**

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**Problem 6.1:** This problem concerns the following circuit.



- (a) Find  $v_r(t)$  for  $t > 0$  if  $i_\ell(0) = 0$ .  
 (b) Find  $v_r(t)$  for  $t > 0$  if  $i_\ell(0) = 5$ .
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**Solution:**

- (a) At  $t = 0$  the inductor looks like an open circuit.

$$v_r(0) = \frac{15}{25} \cdot 10 = 6$$

At  $t = \infty$  the inductor looks like a short circuit.

$$v_r(\infty) = 0.$$

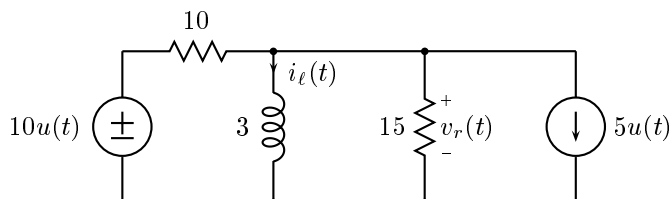
We turn off the source to determine the time constant

$$\tau = \frac{L}{R_{eq}} = \frac{3}{6} = \frac{1}{2}.$$

Therefore,

$$v_r(t) = 6e^{-2t} \quad t \geq 0.$$

- (b) We can incorporate a current source to handle the initial condition. This changes the circuit to



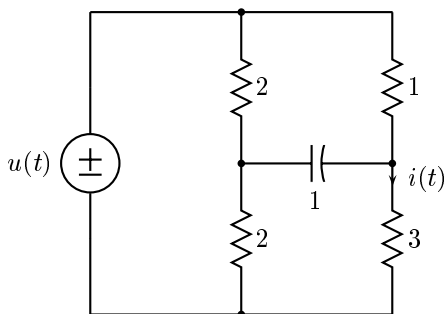
The time constant is the same as before. The value at  $t = \infty$  is also the same as before. At  $t = 0$

$$v_r(0) = 6 - 30 = -24.$$

Therefore,

$$v_r(t) = -24e^{-2t} \quad t \geq 0.$$

**Problem 6.2:** For the circuit below, determine the current  $i(t)$  “by inspection”



**Solution:** At  $t = 0$  the capacitor looks like a short circuit. In this case the upper resistors are in parallel and have an equivalent resistance of  $2/3\Omega$ . The two lower resistors are also in parallel with an effective resistance of  $6/5\Omega$ . Therefore, the voltage across the  $3\Omega$  resistor is (by a voltage divider)

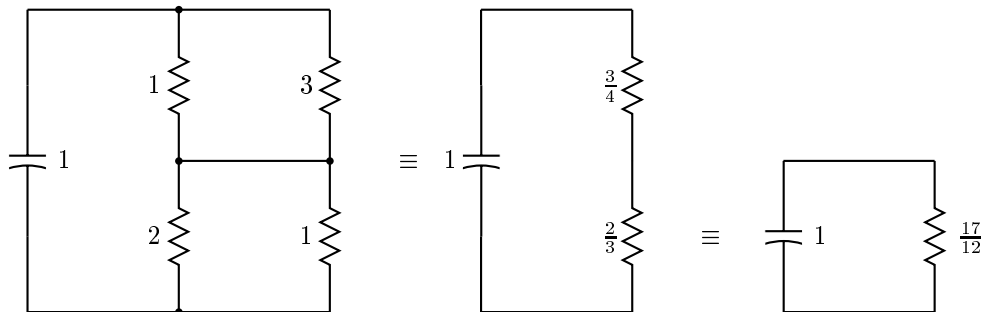
$$\frac{\frac{6}{5}}{\frac{6}{5} + \frac{2}{3}} = \frac{9}{14} \text{ volts.}$$

and  $i(0) = \frac{9}{14} \frac{1}{3} = \frac{3}{14}$  volts.

At  $t = \infty$  the capacitor looks like an open circuit. By a voltage divider, the voltage across the  $3\Omega$  resistor is  $3/4$  volt and the current is  $1/4$  amp.

$$i(\infty) = \frac{1}{4} \text{ amp}$$

When the voltage source is turned off the circuit reduces to

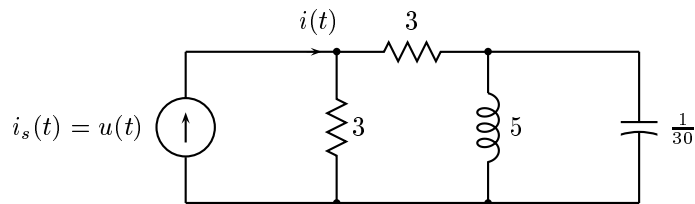


Therefore,  $\tau = RC = \frac{17}{12}$  and

$$i(t) = \frac{1}{4} + \left( \frac{3}{14} - \frac{1}{4} \right) e^{-\frac{12}{17}t} = \frac{1}{4} - \frac{1}{28} e^{-\frac{12}{17}t}$$


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**Problem 6.3:**



- (a) For the circuit above, determine the system function  $H(s)$  that relates the output  $i(t)$  to the input  $i_s(t)$ , i.e. find

$$H(s) = \frac{I(s)}{I_s(s)}$$

- (b) Determine  $i(t)$  for all values of  $t$  if  $i_s(t) = u(t)$ . Assume that the circuit is at initial rest for  $t < 0$ .
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**Solution:**

- (a)

$$H(s) = \frac{3}{6 + \frac{5s \cdot \frac{30}{s}}{5s + \frac{30}{s}}} \cdot \frac{\frac{30}{s}}{5s + \frac{30}{s}} = \frac{3 \cdot \frac{30}{s}}{30s + \frac{180}{s} + 150} = \frac{3}{s^2 + 5s + 6}$$

- (b)

$$I(s) = \frac{1}{s} \cdot \frac{3}{s^2 + 5s + 6} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = \lim_{s \rightarrow 0} \frac{3}{(s+3)(s+2)} = \frac{1}{2}$$

$$B = \lim_{s \rightarrow -2} \frac{3}{s(s+3)} = -\frac{3}{2}$$

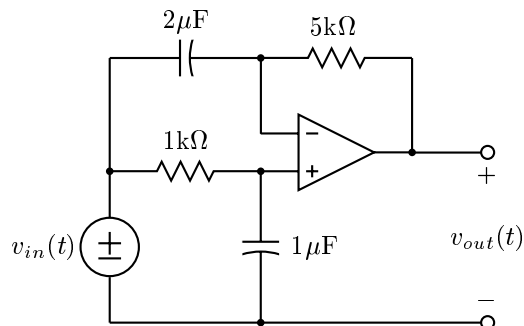
$$C = \lim_{s \rightarrow -3} \frac{3}{s(s+2)} = 1$$

Therefore,

$$i(t) = \left( \frac{1}{2} - \frac{3}{2} e^{-2t} + e^{-3t} \right) u(t)$$


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**Problem 6.4:**



Find the system function  $H(s) = V_{out}(s)/V_{in}(s)$  for the above circuit.

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**Solution:** Let the node potentials at the two nodes connected to the inputs of the opamp be  $E(s)$ . (They are the same.) Then we get for KCL equations

$$C_1 s[E(s) - V_{in}(s)] + \frac{1}{R_2}[E(s) - V_{out}(s)] = 0$$

$$\frac{1}{R_1}[E(s) - V_{in}(s)] + C_2 s E(s) = 0$$

From the second equation

$$E(s) = \frac{1}{(1 + R_1 C_2 s)} V_{in}(s).$$

From the first

$$[1 + C_1 R_2 s]E(s) - R_2 C_1 s V_{in}(s) = V_{out}(s)$$

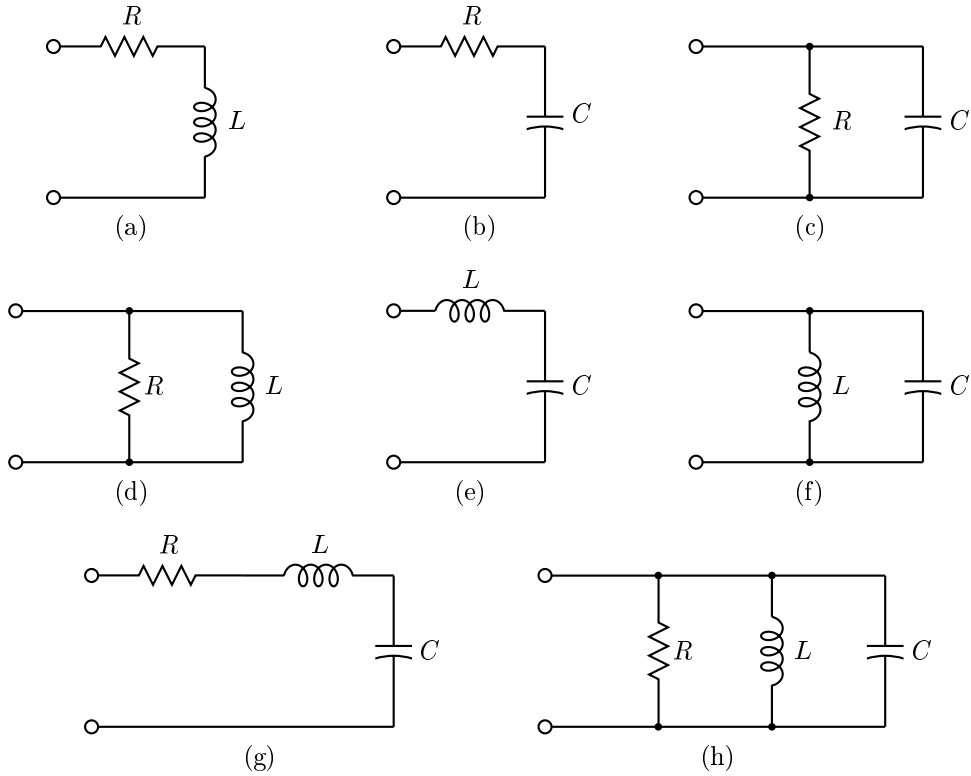
$$\left[ \frac{1 + C_1 R_2 s}{1 + C_2 R_1 s} - R_2 C_1 s \right] V_{in}(s) = V_{out}(s)$$

From this

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1 - C_1 C_2 R_1 R_2 s^2}{1 + C_2 R_1 s} = \frac{1 - 10^{-5} s^2}{1 + 10^{-3} s}$$


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**Problem 6.5:** The *driving-point impedance* (or the equivalent impedance) of a two-terminal network is the ratio of the Laplace transform of its terminal voltage to the Laplace transform of the current entering the positive terminal. For each of the networks below, determine the driving-point impedance. Express your answers as ratios of polynomials in  $s$ .




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**Solution:**

(a)  $Z_a(s) = R + Ls$

(b)  $Z_b(s) = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$

(c)  $Z_c(s) = \frac{\frac{R}{Cs}}{R + \frac{1}{Cs}} = \frac{R}{RCs+1}$

(d)  $Z_d(s) = \frac{RLs}{R+Ls}$

(e)  $Z_e(s) = Ls + \frac{1}{Cs} = \frac{LCs^2+1}{Cs}$

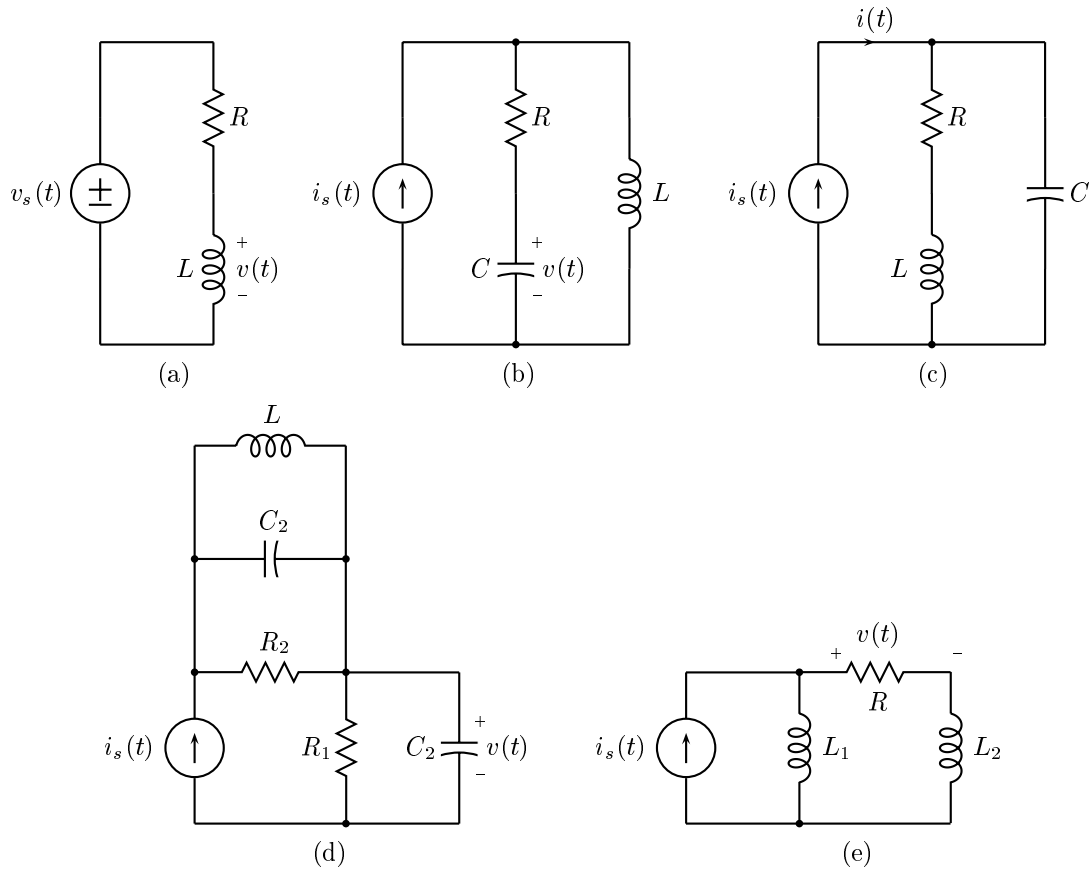
(f)  $Z_f(s) = \frac{\frac{L}{Cs}}{Ls + \frac{1}{Cs}} = \frac{LCs^2+1}{Cs}$

(g)  $Z_g(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs}$

(h)  $Z_h(s) = \frac{1}{\frac{1}{R} + \frac{1}{Ls} + Cs} = \frac{RLs}{RLCs^2+Ls+R}$

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**Problem 6.6:** For each of the networks below determine the system function relating the indicated output variable to the excitation (input). Your answers should be expressed as ratios of polynomials in  $s$ .




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**Solution:**

(a)  $H_a(s) = \frac{Ls}{Ls+R}$

(b)  $H_b(s) = \frac{1}{Cs} \cdot \frac{Ls}{R+\frac{1}{Cs}+Ls} = \frac{Ls}{LCs^2+RCs+1}$

(c)  $H_c(s) = 1$

(d)  $H_d(s) = \frac{1}{C_1s} \cdot \frac{R_1}{R_1+\frac{1}{C_1s}} = \frac{R_1}{R_1C_1s+1}$

(e)  $H_e(s) = \frac{L_1s}{R+(L_1+L_2)s} \cdot R = \frac{RL_1s}{(L_1+L_2)s+R}$

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