

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course EE 2250
Electric Circuit Analysis

January 14, 1999

Problem Set #1—Solutions

Problem 1.1: The voltage waveform for the voltage source in Figure 1 is

$$v_s(t) = \begin{cases} \sin 2\pi(100)t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

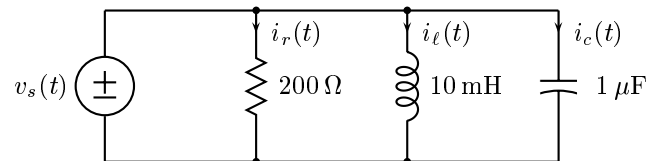


Figure 1:

- (a) Determine $i_r(t)$.
- (b) Determine $i_l(t)$.
- (c) Determine $i_c(t)$.

Solution:

- (a) The voltage drop across all three elements is $v_s(t)$. Thus, for the resistor the current is

$$i_r(t) = \frac{1}{200}v_s(t) = \begin{cases} 0.005 \sin 2\pi(100)t, & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

- (b) The current through the inductor is

$$i_l(t) = \frac{1}{.01} \int_0^t v_s(t)(\beta) d\beta + i_l(0).$$

Assuming that the initial value of the inductor current is zero, this is

$$i_l(t) = \begin{cases} \frac{1}{2\pi} [1 - \cos 2\pi(100)t], & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

- (c) The current through the capacitor is proportional to the derivative of the voltage drop. Thus,

$$i_c(t) = C \frac{dv_s(t)}{dt} = \begin{cases} 2\pi 10^{-4} \cos 2\pi(100)t, & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Problem 1.2: Consider the four terminal network N_1 shown in Figure 2.

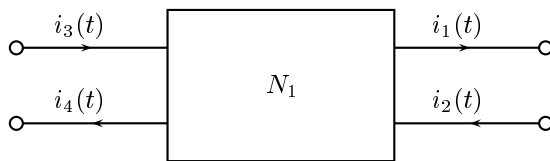


Figure 2:

- (a) When network N_1 is connected to the two subnetworks N_2 and N_3 as shown in Figure 3, what is the relation between currents $i_1(t)$ and $i_2(t)$?

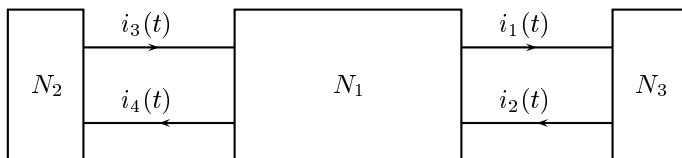


Figure 3:

- (b) Does the result that you derived in (a) apply to $i_1(t)$ and $i_2(t)$ when N_1 is embedded in a larger network as shown in Figure 4? Explain.

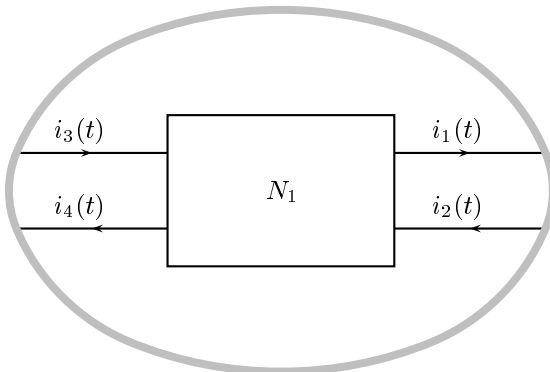


Figure 4:

Solution:

- (a) If we enclose the subnetwork N_2 by a surrounding surface, we can apply KCL and derive the constraint

$$i_3(t) - i_4(t) = 0 \implies i_3(t) = i_4(t).$$

Similarly if we enclose the subnetwork N_3 by such a surface, KCL implies that

$$i_1(t) = i_2(t).$$

- (b) Here the only constraint on the currents is the single KCL equation

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) = 0.$$

$i_1(t)$ is not necessarily equal to $i_2(t)$ and $i_3(t)$ is not necessarily equal to $i_4(t)$.

Problem 1.3: (a) Write the KCL equations that constrain the currents at all of the nodes of the network in Figure 5.

- (b) Write the KVL equations that constrain the voltages for all of the meshes in that same figure.

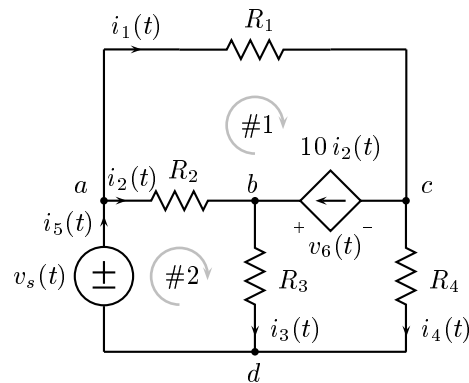


Figure 5:

Solution:

- (a) The network contains four nodes. If we write the KCL equations so that the sum of the currents entering the nodes is zero, then

$$\begin{aligned} \text{node } a: & -i_1(t) - i_2(t) + i_5(t) = 0 \\ \text{node } b: & +i_2(t) + 10i_2(t) - i_3(t) = 0 \\ \text{node } c: & +i_1(t) - 10i_2(t) - i_4(t) = 0 \\ \text{node } d: & +i_3(t) + i_4(t) - i_5(t) = 0 \end{aligned}$$

- (b) Let the resistor voltages be defined consistent with the default sign convention. Then the KVL equations on the three meshes are:

$$\begin{aligned} \text{mesh 1: } & v_1(t) - v_6(t) - v_2(t) = 0 \\ \text{mesh 2: } & -v_s(t) + v_2(t) + v_3(t) = 0 \\ \text{mesh 3: } & -v_3(t) + v_6(t) + v_4(t) = 0 \end{aligned}$$

Problem 1.4: Determine $v(t)$ and $i(t)$ in the network shown in Figure 6

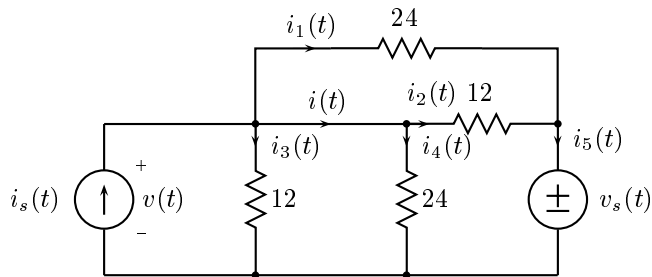


Figure 6:

Solution: The two variables of interest, $v(t)$ and $i(t)$, can readily be expressed in terms of the element variables as

$$\begin{aligned} v(t) &= v_3(t) \\ i(t) &= i_2(t) + i_4(t). \end{aligned}$$

Therefore, it will be sufficient to solve for the resistor voltages and currents. If there are eight variables, we must have eight independent equations. We can get four of these from the element relations for the resistors

$$\begin{aligned} v_1(t) &= 24i_1(t) \\ v_2(t) &= 12i_2(t) \\ v_3(t) &= 12i_3(t) \\ v_4(t) &= 24i_4(t) \end{aligned}$$

one from KCL at the large node at the upper left

$$i_1(t) + i_2(t) + i_3(t) + i_4(t) = i_s(t),$$

and three from KVL equations (ignoring the mesh with the current source)

$$\begin{aligned} v_1(t) - v_2(t) &= 0 \\ v_3(t) - v_4(t) &= 0 \\ -v_2(t) + v_4(t) &= v_s(t). \end{aligned}$$

The four element relations will allow us to express the resistor voltages in terms of their currents. Furthermore, the first two KVL equations allow us to eliminate two of the currents since

$$\begin{aligned}v_1(t) = v_2(t) &\implies 2i_1(t) = i_2(t) \\v_3(t) = v_4(t) &\implies 2i_4(t) = i_3(t)\end{aligned}$$

Thus the KCL equation reduces to

$$i_1(t) + i_4(t) = \frac{1}{3}i_s(t)$$

and the remaining KVL equation becomes

$$-i_1(t) + i_4(t) = \frac{1}{24}v_s(t).$$

These final equations are straightforward to solve. Adding the two equations gives

$$i_4(t) = \frac{1}{6}i_s(t) + \frac{1}{48}v_s(t).$$

Substituting this result into the first of these equations gives

$$i_1(t) = \frac{1}{6}i_s(t) - \frac{1}{48}v_s(t).$$

Since $2i_4(t) = i_3(t)$, we have

$$v(t) = v_3(t) = 24i_4(t) = 4i_s(t) + \frac{1}{2}v_s(t)$$

and

$$i(t) = i_2(t) + i_4(t) = 2i_1(t) + i_4(t) = \frac{1}{2}i_s(t) - \frac{1}{48}v_s(t).$$

Problem 1.5: In the center of Figure 7 is a model of a one-transistor preamplifier that is used to amplify the output of a low amplitude magnetic pickup, and drive a $25\text{ k}\Omega$ load. Express the voltage $v_L(t)$ measured across the load in terms of $v_s(t)$.

Solution: We begin by writing a KVL equation around the left mesh. This will allow us to solve for $i_1(t)$ in terms of $v_s(t)$.

$$v_s(t) = 2000i_1(t) + 8000i_1(t)$$

or

$$i_1(t) = \frac{1}{10,000}v_s(t).$$

The voltage drop across the $100\text{ k}\Omega$ resistor is $v_L(t)$. Writing a KCL equation at the node connected to the $+$ terminal of the load gives

$$\frac{1}{25,000}v_L(t) + \frac{1}{100,000}v_L(t) = -100i_1(t).$$

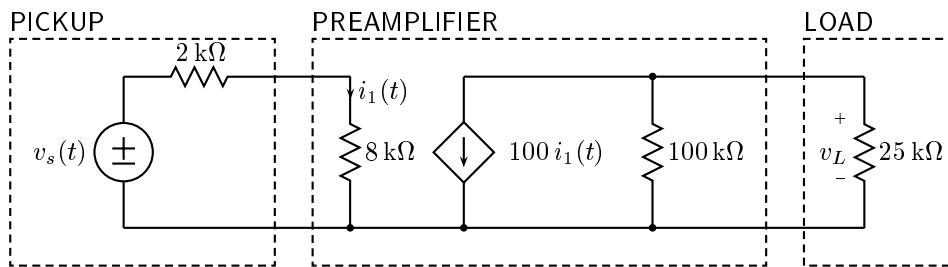


Figure 7:

From this we deduce

$$v_L(t) = -2,000,000i_1(t) = -200v_s(t).$$
