GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

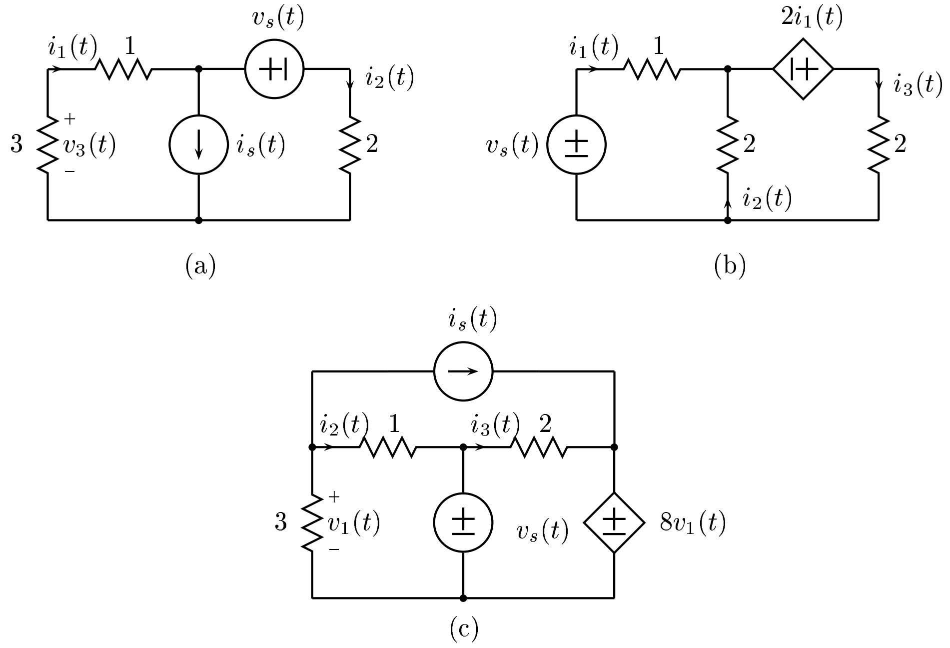
Course EE 2250

Electric Circuit Analysis

January 21, 1999

Problem Set #2—Solutions

Problem 2.1: This problem is concerned with the three networks shown in Figure 1.



(a) (b)

(c)

Figure 1: Circuits for Problem 2.1.

(a) For the network in (a)
  (i.) Draw the basic network.
  (ii.) Identify the closed paths in the original network that correspond to meshes in the basic network.
  (iii.) Identify the closed surfaces in the original network that correspond to nodes in the basic network.
(b) Repeat for the network in (b).
(c) Repeat for the network in (c).

Solution:

(a) (i) The basic network is shown in Figure 2.

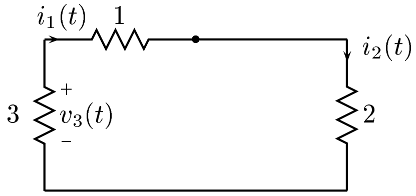


Figure 2: The basic network for the circuit in Figure 1(a).

(ii) There is only one mesh in the basic network and it corresponds to the path around the outside of the original network, i.e., it is the path that contains both the 3Ω and the 2Ω resistors.

(iii) The surfaces in the complete network that correspond to nodes in the basic network are indicated in Figure 3 as dashed lines.

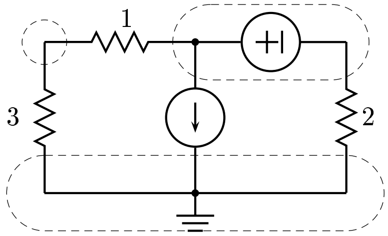


Figure 3: The surfaces corresponding to nodes in the basic network of Figure 2.

(b) (i) The basic network is shown in Figure 4.

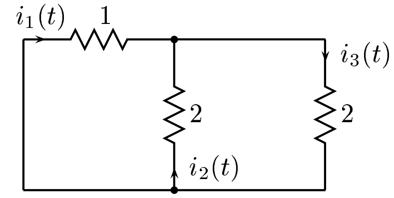


Figure 4: The basic network for the circuit in Figure 1(b).

(ii) The basic network contains two meshes. The paths corresponding to those meshes in the complete network are the two meshes in the complete network.

(iii) The basic network contains two nodes. In the complete network, those correspond to the two closed surfaces illustrated in Figure 5.

(c) (i) The basic network is shown in Figure 6.

(ii) The meshes in the basic network correspond to the two lower meshes in the complete network.

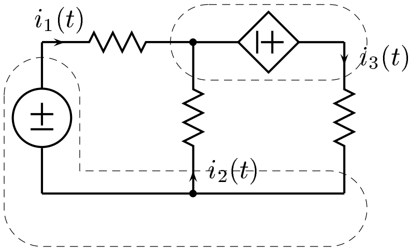


Figure 5: The surfaces corresponding to nodes in the basic network of Figure 4.

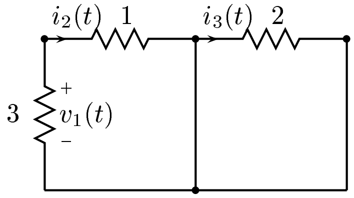


Figure 6: The basic network for the circuit in Figure 1(c).

(iii) The basic network contains only two nodes. In the complete network these correspond to the dashed surfaces shown in Figure 7. Since both terminals of the 2Ω resistor are included in that nose, the resistor itself is incorporated in it.

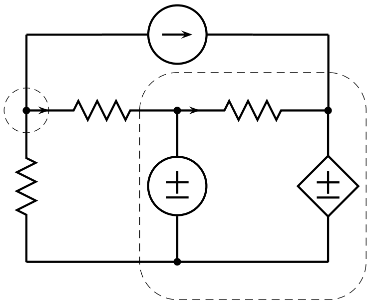


Figure 7: The surfaces corresponding to nodes in the basic network of Figure 4.

Problem 2.2: Use the node method to write a set of equilibrium equations for the network of Figure 8. Use as variables the voltages $e_a(t)$, $e_b(t)$, and $e_c(t)$. Do not solve the set of equations.

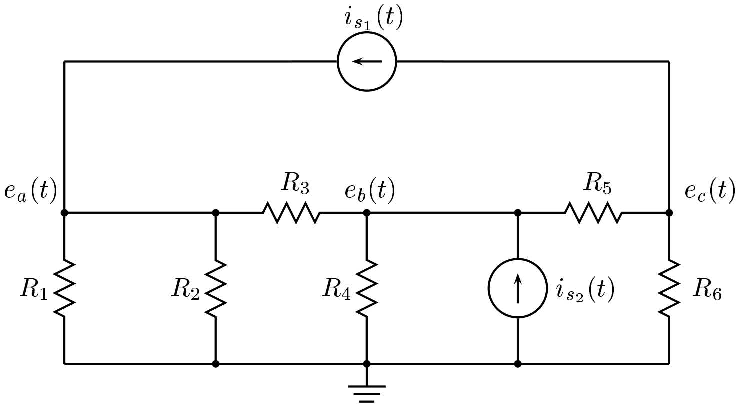


Figure 8: Circuit for Problem 2.2.

Solution:

$$\begin{aligned}
\text{node } a: &\quad \frac{1}{R_1}e_a(t) + \frac{1}{R_2}e_a(t) + \frac{1}{R_3}[e_a(t) - e_b(t)] = i_{s_1}(t) \\
\text{node } b: &\quad \frac{1}{R_3}[e_b(t) - e_a(t)] + \frac{1}{R_4}e_b(t) + \frac{1}{R_5}[e_b(t) - e_c(t)] = i_{s_2}(t) \\
\text{node } c: &\quad \frac{1}{R_5}[e_c(t) - e_b(t)] + \frac{1}{R_6}e_c(t) = -i_{s_1}(t)
\end{aligned}$$

Regrouping terms allows us to write these in the alternative form

$$\begin{aligned}
\text{node } a: &\quad \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] e_a(t) - \frac{1}{R_3}e_b(t)] = i_{s_1}(t) \\
\text{node } b: &\quad -\frac{1}{R_3}e_a(t) + \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] e_b(t) - \frac{1}{R_5}e_c(t) = i_{s_2}(t) \\
\text{node } c: &\quad -\frac{1}{R_5}e_b(t) + \left[\frac{1}{R_5} + \frac{1}{R_6}\right] e_c(t) = -i_{s_1}(t)
\end{aligned}$$

Problem 2.3: Determine the voltage at each connection point in the circuit in Figure 9 with respect to the indicated ground.

Solution: We need to write a KCL equation at each of the three nodes of the basic network, a, b, and c. The node potential $e_d(t)$ is equal to $-v_s(t)$.

$$\begin{aligned}
\text{node } a: &\quad [e_a(t) - e_b(t)] + e_a(t) + [e_a(t) + v_s(t)] = 0 \\
\text{node } b: &\quad [e_b(t) - e_a(t)] + e_b(t) + [e_b(t) - e_c(t)] = +i_s(t) \\
\text{node } c: &\quad [e_c(t) - e_b(t)] + e_c(t) + [e_c(t) + v_s(t)] = i_s(t)
\end{aligned}$$

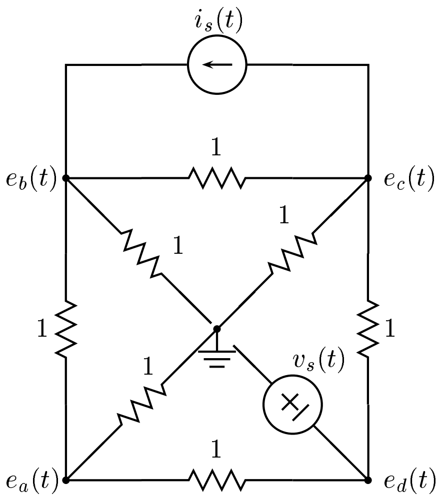


Figure 9: Circuit for Problem 2.3

We can combine the three equations into one matrix-vector equation

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} e_s(t) \\ e_b(t) \\ e_c(t) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} v_s(t) + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} i_s(t).$$

The solution is

$$\begin{aligned} e_a(t) &= -0.4286\, v_s(t) + 0.0952\, i_s(t) \\ e_b(t) &= -0.2857\, v_s(t) - 0.2857\, i_s(t) \\ e_c(t) &= -0.4286\, v_s(t) - 0.2381\, i_s(t). \end{aligned}$$

Problem 2.4: Find all of the element voltages and currents in the circuit of Figure 10 using the mesh method. Be sure to identify the variables clearly.

Solution: The first step is to identify the meshes in the basic network and to identify the variables in the circuit. This is done in Figure 11. The voltages are implied by the currents using the default sign convention. The two mesh equations are

$$\begin{aligned} \text{mesh } \alpha: &\quad 2[i_\alpha(t) - i_s(t)] + i_\alpha(t) + 2[i_\alpha - i_\beta] + v_s(t) = 0 \\ \text{mesh } \beta: &\quad -v_s(t) + 2[i_\beta(t) - i_\alpha(t)] + i_\beta(t) + i_\beta(t) = 0. \end{aligned}$$

These can be put into matrix-vector form as:

$$\begin{bmatrix} 5 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} i_s(t) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} v_s(t)$$

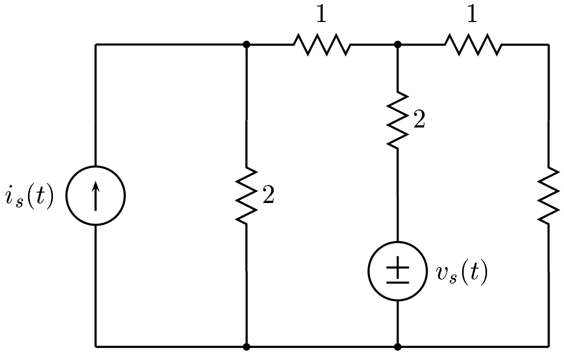


Figure 10: Circuit for Problem 2.4.

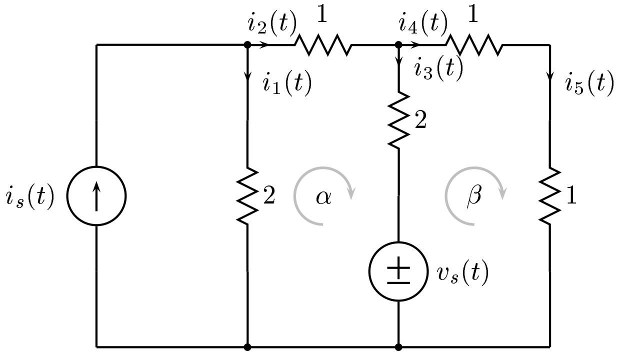


Figure 11: Circuit for Problem 2.4 with the meshes indicated and the currents defined.

The solution is

$$
\begin{aligned}
i_\alpha(t) &= \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t) \\
i_\beta(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t).
\end{aligned}
$$

From these we can compute all of the element variables.

$$
\begin{aligned}
i_1(t) &= \frac{1}{2}i_s(t) + \frac{1}{8}v_s(t) \\
i_2(t) &= \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t) \\
i_3(t) &= \frac{1}{4}i_s(t) - \frac{5}{16}v_s(t) \\
i_4(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t) \\
i_5(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t) \\
v_1(t) &= i_s(t) + \frac{1}{4}v_s(t)
\end{aligned}
$$

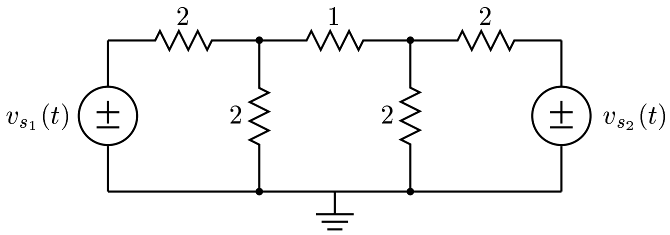


Figure 12: Circuit for Problem 2.5.

$$\begin{aligned}
v_2(t) &= \frac{1}{2}i_s(t) - \frac{1}{8}v_s(t)\\
v_3(t) &= \frac{1}{2}i_s(t) - \frac{5}{8}v_s(t)\\
v_4(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t)\\
v_5(t) &= \frac{1}{4}i_s(t) + \frac{3}{16}v_s(t)
\end{aligned}$$

Problem 2.5: In our derivation of the mesh method, we stressed its duality with the node method, i.e., the similarity of the two methods if the roles of voltages and currents, and nodes and meshes, are reversed. This problem lets you explore this issue further. Consider the network in Figure 12.

(a) Use the node method to determine the set equations that must be solved to find the equilibrium solution. Omit the ground node when writing your equations. Express these equations in the form

$$\boldsymbol{C}\boldsymbol{v}(t) = \boldsymbol{s}_1 v_{s_1}(t) + \boldsymbol{s}_2 v_{s_2}(t).$$

Here $\boldsymbol{v}(t)$ is a vector of node potentials, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are column vectors of constants, and $\boldsymbol{C}$ is a constant matrix.

(b) Now design a *different* network containing two *current* sources with currents $i_{s_1}(t)$ and $i_{s_2}(t)$, such that the set of *mesh* equations that need to be solved to find the equilibrium solution is

$$\boldsymbol{C}\boldsymbol{i}(t) = \boldsymbol{s}_1 i_{s_1}(t) + \boldsymbol{s}_2 i_{s_2}(t).$$

and $\boldsymbol{i}(t)$ is the vector of mesh currents.

(c) Solve your equations from part (b).

Solution:

(a) There are two nodes, which are located at the terminals of the 1Ω resistor. This leads to the two equations

$$\begin{aligned}
\text{node } a: &\quad \frac{1}{2}[e_a(t) - v_{s_1}(t)] + \frac{1}{2}e_a(t) + [e_a(t) - e_b(t)] = 0\\
\text{node } b: &\quad [e_b(t) - e_a(t)] + \frac{1}{2}e_b(t) + \frac{1}{2}[e_b(t) - v_{s_2}(t)] = 0
\end{aligned}$$

These can be written in matrix-vector form as

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} v_{s_1}(t) + \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} v_{s_2}(t).$$

(b) The mesh equations for this circuit should look like the node equations for the previous one

$$\begin{aligned} \text{mesh } a\text{:} \quad & \frac{1}{2}[i_a(t) - i_{s_1}(t)] + \frac{1}{2}i_a(t) + [i_a(t) - i_b(t)] = 0 \\ \text{mesh } b\text{:} \quad & [i_b(t) - i_a(t)] + \frac{1}{2}i_b(t) + \frac{1}{2}[i_b(t) - i_{s_2}(t)] = 0 \end{aligned}$$

These are mesh equations for the circuit drawn in Figure 13.

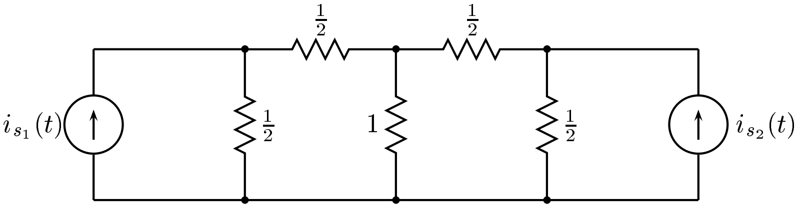


Figure 13: The dual to the circuit in Figure 12.

(c)

$$\begin{aligned} i_a(t) &= \frac{1}{3}i_{s_1}(t) + \frac{1}{6}i_{s_2}(t) \\ i_b(t) &= \frac{1}{6}i_{s_1}(t) + \frac{1}{3}i_{s_2}(t) \end{aligned}$$
