GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

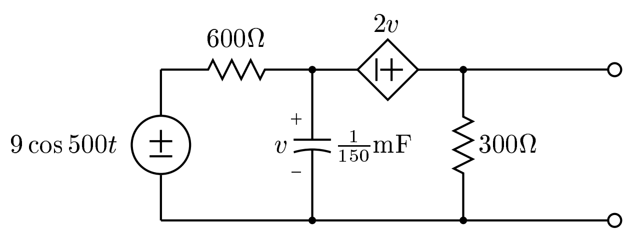
Course EE 2250

Electric Circuit Analysis

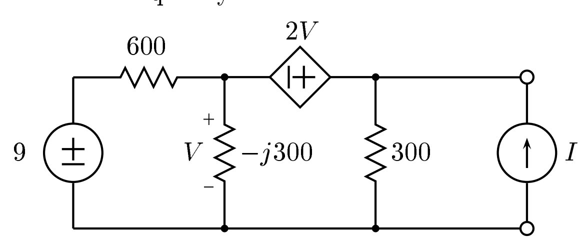
February 11, 1999

Problem Set #5–Solutions

Problem 5.1: Find the Thévenin equivalent circuit for the circuit shown in the figure below using the mesh current method.



Solution: We can simplify this problem considerably if we acknowledge the fact that the only frequency of interest is 500 rad/sec. Then we can replace each element by its impedance at that frequency.



To find the Thévenin equivalent we shall attach a current source with a complex current amplitude I and solve for the complex voltage amplitude V_o. Since

$$V_o = V_T + Z_T I,$$

we can get the Thévenin equivalent network from these parameters.

Let I_α and I_β be the complex amplitudes of the two mesh currents associated with the left and center meshes. Note that $V = j300(I_\beta - I_alpha)$. The two mesh equations are

$$600I_\alpha - j300(I_\alpha - I_\beta) = 9$$
$$-j900(I_\alpha - I_\beta) + 300(I_\beta + I) = 0.$$

These reduce to

$$(600 - j300)I_\alpha + j300I_\beta = 9$$
$$-j900I_\alpha + (300 - j900)I_\beta = -300I.$$

Using MATLAB we get

$$I_\alpha = (0.0130 + j0.0006) + (-0.1321 + j0.0377)I$$
$$I_\beta = (0.0119 - j0.0034) + (-0.2075 - j0.2264)I$$

Since $V_o = 300(I + I_\beta)$, we have

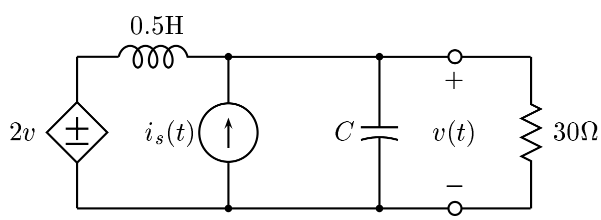
$$V_o = (237.75 - j67.92)I + (3.57 + j1.02)$$

and

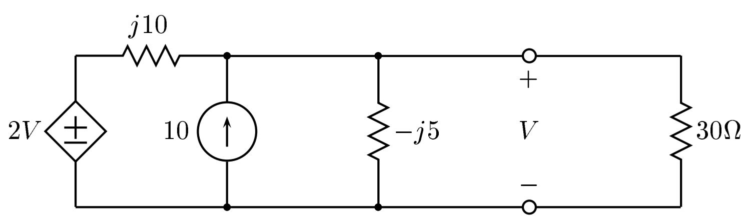
$$Z_T = 237.75 - j67.92$$
$$V_T = 3.57 + j1.02$$

at $\omega = 500$ rad/s.

Problem 5.2: Find $v(t)$ by first replacing the circuit to the left of the terminals with its Thévenin equivalent circuit when $C = 10$ mF. The current in the current source is $i_s(t) = 10\cos 20t$.



Solution: Let us begin by replacing each of the elements by its equivalent impedance.



If I denotes the current entering the subnetwork from the resistor, then a KCL equation written at the node that includes the + terminal is

$$\frac{2V - V}{j10} + 10 + \frac{V}{j5} + I = 0$$

or

$$j\frac{3}{10}V = I + 10.$$

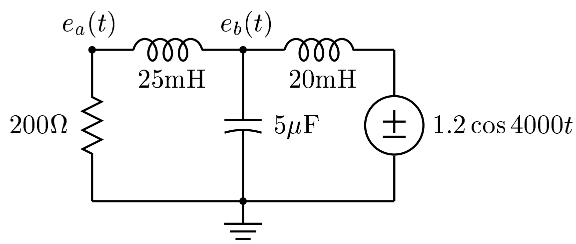
From the $v - i$ relation for the 30Ω resistor, however, $I = -V/30$. Thus,

$$j\frac{3}{10}V = -\frac{1}{30}V + 10$$
$$V = \frac{300}{1 - j9} = 33.1295e^{j1.4601}$$

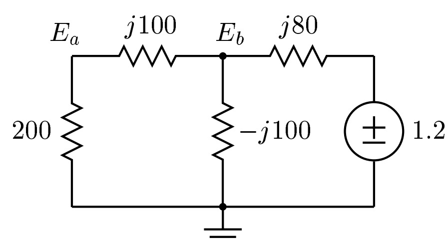
Thus,

$$v(t) = 33.1295\cos(20t + 1.4601)$$

Problem 5.3: Find the two node voltages $e_a(t)$ and $e_b(t)$ in the circuit below.



Solution: Here we can replace each element by its impedance at 4000 rad/s. This allows us to redraw the circuit at



The two node equations are:

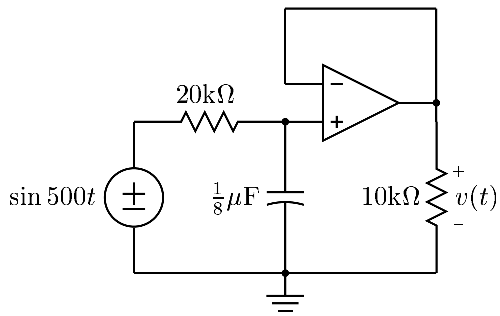
$$\frac{E_a}{200} + \frac{E_a - E_b}{j100} = 0$$
$$\frac{E_b - E_a}{j100} - \frac{E_b}{j100} + \frac{E_b - 1.2}{j80} = 0$$

These can be reduced to

$$(2 + j)E_a - 2E_b = 0$$
$$-E_a + 1.25E_b = 1.5$$

Solving these gives $E_a = 0.8276 - j2.0690$, $E_b = 1.8621 - j1.6552$.

Problem 5.4: For the circuit below find $v(t)$.



Solution: The opamp acts like a voltage follower, i.e., $v(t)$ is equal to the voltage across the capacitor. Furthermore, we know that $v(t)$ is of the form

$$v(t) = \Im\text{m}\left[Ve^{j500t}\right].$$

To determine V, replace the source by a complex exponential time function, the elements by their equivalent impedances, and use the voltage divider.

$$V = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}.$$

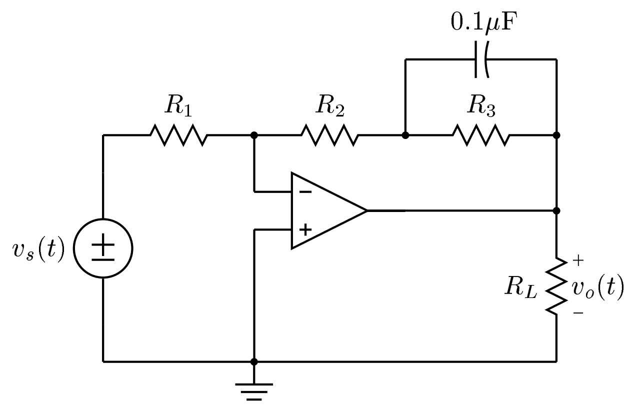
Letting $R = 20 \times 10^3$; $C = 0.125 \times 10^{-6}$, $\omega = 500$, we get

$$\begin{aligned} V &= \frac{1}{1 + j(20 \times 10^3)(0.125 \times 10^{-6})(500)} = \frac{1}{1 + j(1.25)} \\ &= 0.625 e^{-j \tan^{-1} 1.25} \\ v(t) &= 0.625 \sin(500t - \tan^{-1} 1.25). \end{aligned}$$

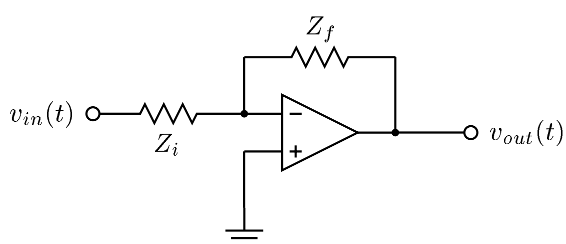
Problem 5.5: In the circuit below the value of R_1 is 10kΩ.

(a) Determine the values of R_2 and R_3 so that the gain at low frequencies is 5 and the gain at high frequencies is 2.

(b) Determine the frequency at which the gain is midway between these two values, i.e. the frequency at which the gain is 3.5.



Solution:
(a) This circuit is a special case of an inverting amplifier as shown below.



The frequency response of such a system is known to be

$$H(j\omega) = -\frac{Z_f(j\omega)}{Z_i(j\omega)}.$$

For this circuit, the input impedance is simply

$$Z_i = R_1$$

and the other three elements contribute to the feedback impedance

$$\begin{aligned} Z_f &= R_2 + \frac{R_3 \frac{1}{j\omega C}}{R_3 + \frac{1}{j\omega C}} = R_2 + \frac{R_3}{1 + j\omega R_3 C} \\ &= \frac{(R_2 + R_3) + j\omega R_2 R_3 C}{1 + j\omega R_3 C}. \end{aligned}$$

Therefore,

$$H(j\omega) = -\frac{(R_2 + R_3) + j\omega R_2 R_3 C}{R_1(1 + j\omega R_3 C)}.$$

To measure the low frequency gain, we can let $\omega = 0$ and take the magnitude

$$G_{LF} = \frac{R_2 + R_3}{R_1}.$$

To measure the high frequency gain, we can take the (magnitude of the) limit as ω grows large

$$G_{HF} = \frac{R_2}{R_1}.$$

Since $R_1 = 10\text{k}\Omega$ and we want $G_{LF} = 5$ and $G_{HF} = 2$, we can substitute and solve for R_2 and R_3.

$$5 = \frac{R_2 + R_3}{10k\Omega} \Longrightarrow R_2 + R_3 = 50k\Omega$$
$$2 = \frac{R_2}{10k\Omega} \Longrightarrow R_2 = 20k\Omega$$

Therefore, we want to choose $R_2 = 20k\Omega$ and $R_3 = 30k\Omega$.

(b) With the values substituted, the frequency response is

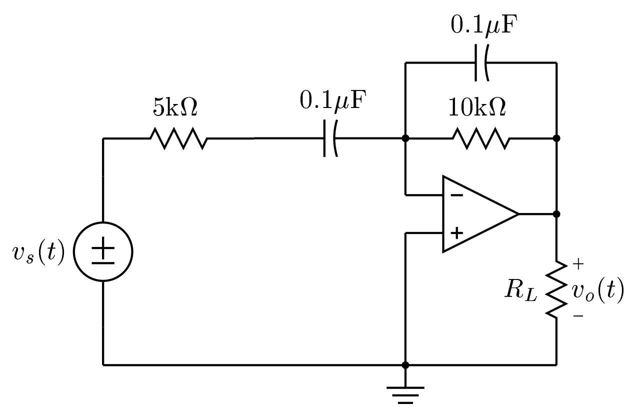
$$H(j\omega) = -\frac{5 + j\omega(0.6)}{1 + j\omega(0.3)}.$$

To find the value of ω for which the magnitude is 3.5, we solve the equation

$$\frac{25 + \omega^2(0.6)^2}{1 + \omega^2(0.3)^2} = (3.5)^2.$$

This gives $\omega = 4.14$.

Problem 5.6: Sketch the asymptotes for the magnitude Bode plot of the frequency response of the circuit below.



Solution: This circuit also has the form of an inverting amplifier

$$H(j\omega) = -\frac{Z_f(j\omega)}{Z_i(j\omega)} = -\frac{\frac{R_2 \frac{1}{jC_2\omega}}{R_2 + \frac{1}{jC_2\omega}}}{R_1 + \frac{1}{jC_1\omega}}$$
$$= -\frac{\frac{jR_2C_1\omega}{1+jR_2C_2\omega}}{1 + jR_1C_1\omega}$$
$$= -\frac{jR_2C_1\omega}{(1 + jR_2C_2\omega)(1 + jR_1C_1\omega)}.$$

But

$$R_1 C_1 = (5000)(10^{-7}) = 5 \times 10^{-4}$$
$$R_2 C_2 = (10000)(10^{-7}) = 10^{-3} = R_2 C_1$$

Therefore,

$$20 \log_{10} |H(j\omega)| = 20 \log_{10}(10^{-3}\omega) - 20 \log_{10} |1 + j(5 \times 10^{-4})\omega| - 20 \log_{10} |1 + j(10^{-3})\omega|$$

The Bode magnitude plot is seen to consist of the sum of three terms.

The first term, $20 \log_{10}(10^{-3}\omega)$ increases at 20 dB/decade for all frequencies. Its value is 0 at $\omega = 1000$ rad/s.

The second term, $-20 \log_{10} |1 + j(5 \times 10^{-4})\omega|$ has a value that is nearly constant at 0 dB for $\omega < 2000$ rad/s, and it decreases at 20 dB/decade for frequencies above that.

The third term is similar to the second except that it has its break at 1000 rad/s.

Adding these three terms together gives a Bode plot that increases with a slope of 20 dB/decade until $\omega = 1000$ rad/s, where it has a value of 0 dB, then is constant until $\omega = 2000$ rad/s, then decreases with a slope of -20 dB/decade for higher frequencies.