GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course EE 2250

Electric Circuit Analysis

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Problem Set #6—Solutions

Problem 6.1: Determine the Laplace transforms of the following time waveforms.

(a) $x_a(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$

(b) $x_b(t) = t^2 e^{-3t}, \;\; t > 0$

(c) $x_c(t) = e^{-4t} \sin 5t, \;\; t > 0$

(d) $x_d(t) = t, \;\; t > 0$

Solution:
(a)

$$X_a(s) = \int_0^T 1 \cdot e^{-st}\, dt = \frac{1}{s}\left(1 - e^{-sT}\right)$$

(b) To get $X_b(s)$ we can use an indirect approach (or we could just do the integral).

$$\begin{aligned} z(t) = e^{-3t} \quad &\longleftrightarrow \quad \frac{1}{s+3} \\ y(t) = te^{-3t} \quad &\longleftrightarrow \quad -\frac{d}{ds}\left(\frac{1}{s+3}\right) = \frac{1}{(s+3)^2} \\ x_b(t) = ty(t) \quad &\longleftrightarrow \quad -\frac{d}{ds}\left(\frac{1}{(s+3)^2}\right) = \frac{2}{(s+3)^3} \end{aligned}$$

(c)

$$\begin{aligned} x_c(t) &= e^{-4t}\sin 5t = \frac{1}{j2}\left(e^{-4t}e^{j5t} - e^{-4t}e^{-j5t}\right) \\ &= \frac{1}{j2}e^{-(4-j5)t} - \frac{1}{j2}e^{-(4+j5)t} \\ X_c(s) &= \frac{\frac{1}{j2}}{s+4-j5} - \frac{\frac{1}{j2}}{s+4+j5} \\ &= \frac{5}{s^2+8s+41} \end{aligned}$$

(d)

$$\begin{aligned} x_d(t) &= t = 1 \cdot t \\ X_d(s) &= -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2} \end{aligned}$$

Problem 6.2: Find the inverse Laplace transform of

$$X(s) = \frac{2s+6}{s(s^2+3s+2)}.$$

Solution:

$$\begin{aligned} X(s) &= \frac{2s+6}{s(s^2+3s+2)} = \frac{2s+6}{s(s+1)(s+2)} \\ &= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} \end{aligned}$$

We can evaluate A, B, and C using

$$\begin{aligned} A &= \lim_{s\to 0} \frac{2s+6}{(s+1)(s+2)} = 3 \\ B &= \lim_{s\to -1} \frac{2s+6}{s(s+2)} = -4 \\ C &= \lim_{s\to -2} \frac{2s+6}{s(s+1)} = 1 \end{aligned}$$

Therefore,

$$x(t) = 3 - 4e^{-t} + e^{-2t}, \quad t > 0.$$

Problem 6.3: Find the inverse Laplace transform of

$$X(s) = \frac{s+3}{s^3+3s^2+6s+4}.$$

Solution:

$$\begin{aligned} X(s) &= \frac{s+3}{s^3+3s^2+6s+4} = \frac{s+3}{(s+1)(s^2+2s+4)} \\ &= \frac{s+3}{(s+1)(s+1-j\sqrt{3})(s+1+j\sqrt{3})} \\ &= \frac{A}{s+1} + \frac{B}{s+1-j\sqrt{3}} + \frac{B^*}{s+1+j\sqrt{3}}. \end{aligned}$$

$$\begin{aligned} A &= \lim_{s\to -1} \frac{s+3}{s^2+2s+4} = \frac{2}{3} \\ B &= \lim_{s\to(-1+j\sqrt{3})} \frac{s+3}{(s+1)(s+1+j\sqrt{3})} = -\frac{1}{3} - j\frac{1}{2\sqrt{3}} \end{aligned}$$

Therefore,

$$\begin{aligned} x(t) &= \frac{2}{3}e^{-t} + \left(-\frac{1}{3} - j\frac{1}{2\sqrt{3}}\right) e^{-t}e^{j\sqrt{3}t} + \left(-\frac{1}{3} + j\frac{1}{2\sqrt{3}}\right) e^{-t}e^{-j\sqrt{3}t} \\ &= \frac{2}{3}e^{-t} - \frac{1}{3}e^{-t}\left(e^{j\sqrt{3}t} + e^{-j\sqrt{3}t}\right) + \frac{1}{j2\sqrt{3}}e^{-t}\left(e^{j\sqrt{3}t} - e^{-j\sqrt{3}t}\right) \\ &= \frac{2}{3}e^{-t} - \frac{2}{3}e^{-t}\cos\sqrt{3}t + \frac{1}{\sqrt{3}}e^{-t}\sin\sqrt{3}t, \quad t > 0 \end{aligned}$$

Problem 6.4: Find the inverse Laplace transform of

$$X(s) = \frac{5s-1}{s^3 - 3s - 2}.$$

Solution:

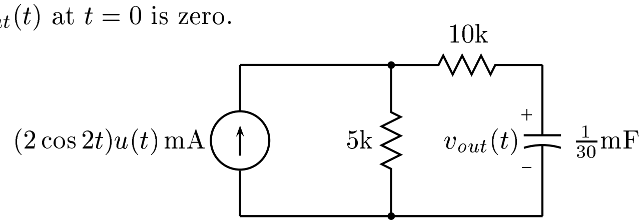
$$\begin{aligned} X(s) &= \frac{5s-1}{s^3 - 3s + 2} = \frac{5s-1}{(s+1)^2(s-2)} \\ &= \frac{A}{s-2} + \frac{B}{s+1} + \frac{C}{(s+1)^2} \end{aligned}$$

$$\begin{aligned} A &= \lim_{s\to 2} \frac{5s-1}{(s+1)^2} = 1 \\ B &= \lim_{s\to -1} \frac{d}{ds}\left(\frac{5s-1}{s-2}\right) = \lim_{s\to -1} \frac{(s-2)5 - (5s-1)}{(s-2)^2} = -1 \\ C &= \lim_{s\to -1} \frac{5s-1}{s-2} = 2 \end{aligned}$$

Therefore,

$$x(t) = e^{-2t} - e^{-t} + 2te^{-t} = e^{-2t} + (2t-1)e^{-t}, \quad t > 0.$$

Problem 6.5: Using the Laplace transform, find $v_{out}(t)$ for $t > 0$ for the circuit shown below. The initial value of $v_{out}(t)$ at $t = 0$ is zero.



Solution: Writing a KVL equation around the outer loop gives (currents in milliamps, resistances in kilohms, voltages in volts, capacitances in millifarads)

$$v_{out}(t) + 15i(t) = 10\cos 2t.$$

But we know

$$i(t) = \frac{1}{30}\frac{dv_{out}(t)}{dt}$$

Therefore,

$$v_{out}(t) + \frac{1}{2}\frac{dv_{out}(t)}{dt} = 10\cos 2t$$

or

$$\frac{dv_{out}(t)}{dt} + 2v_{out}(t) = 20\cos 2t$$

Taking Laplace transforms

$$[sV_{out}(s) - v_{out}(0)] + 2V_{out}(s) = \frac{20s}{s^2+4}$$

$$\begin{aligned} V_{out}(s) &= \frac{20s}{(s+2)(s^2+4)} = \frac{20s}{(s+2)(s-j2)(s+j2)} \\ &= \frac{A}{s+2} + \frac{B}{s-j2} + \frac{B^*}{s+j2} \end{aligned}$$

$$\begin{aligned} A &= \lim_{s\to -2}\frac{20s}{s^2+4} = -5 \\ B &= \lim_{s\to j2}\frac{20s}{(s+2)(s+j2)} = \frac{5}{2}(1-j) \end{aligned}$$

$$\begin{aligned} x(t) &= -5e^{-2t} + \frac{5}{2}(1-j)e^{j2t} + \frac{5}{2}(1+j)e^{-j2t} \\ &= -5e^{-2t} + 5\cos 2t + 5\sin 2t, \quad t > 0 \end{aligned}$$