GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

**Course EE 2250**

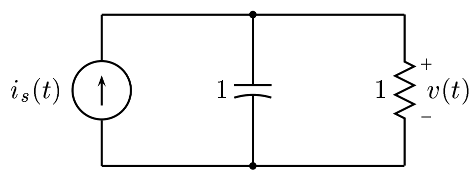
**Electric Circuit Analysis**

February 26, 1999

## Problem Set #7—Solutions

---

**Problem 7.1:** The circuit below is initially at rest (i.e., the initial capacitor voltage is zero.




Determine the voltage $v(t)$ for each of the excitations below:

(a) $i_s(t) = u(t)$.
(b) $i_s(t) = \sin(t)\, u(t)$.
(c) $i_s(t) = tu(t)$.

---

**Solution:**

$$H(s) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s+1}$$

(a)

$$V_a(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

Therefore,

$$v_a(t) = 1 - e^{-t}, \;\; t > 0.$$

(b)

$$V_b(s) = \frac{1}{(s^2+1)(s+1)} = \frac{\frac{1}{2}}{s+1} + \frac{\frac{-1+j}{4}}{s-j} + \frac{\frac{-1-j}{4}}{s+j}$$

Therefore,

$$v_b(t) = \frac{1}{2}e^{-t} - \frac{1}{2}\cos t - \frac{1}{2}\sin t.$$

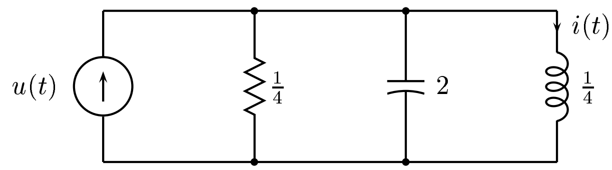
(c)

$$V_c(s) = \frac{1}{s^2(s+1)} = \frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2}$$

Therefore,

$$v_c(t) = t - 1 + e^{-t}, \;\; t > 0.$$

**Problem 7.2:** The circuit below is at rest for $t < 0$. Determine $i(t)$.




**Solution:**

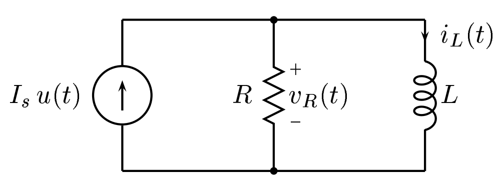
$$I(s) = \frac{1}{s}\left(\frac{\frac{4}{s}}{\frac{4}{s}+4+2s}\right) = \frac{1}{s}\left(\frac{2}{s^2+2s+2}\right)$$
$$= \frac{A}{s} + \frac{B}{s+1-j} + \frac{B^*}{s+1+j}$$

$$A = \lim_{s\to 0}\frac{2}{s^2+2s+2} = 1$$
$$B = \lim_{s\to -1+j}\frac{2}{s(s+1+j)} = -\frac{1}{2}+j$$

Therefore,

$$i(t) = 1 + (-\frac{1}{2}+j)e^{-t}e^{jt} + (-\frac{1}{2}-j)e^{-t}e^{-jt}$$
$$= 1 - \frac{1}{2}e^{-t}(e^{jt}+e^{-jt}) + je^{-t}(e^{jt}-e^{-jt})$$
$$= 1 - e^{-t}\cos t - e^{-t}\sin t, \quad t > 0.$$

**Problem 7.3:** The circuit below is initially at rest. Determine $v_R(t)$ and $i_L(t)$ "by inspection".




**Solution:** At $t = 0$ the inductor is an open circuit.

$$i_L(0) = 0$$
$$v_R(0) = I_sR.$$

At $t = \infty$ the inductor is a short circuit.

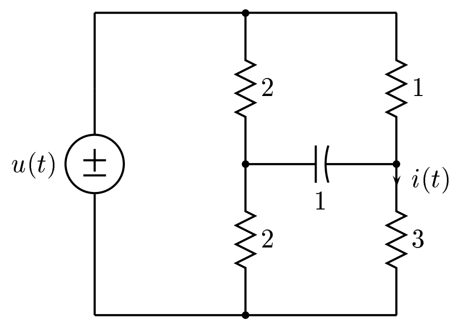
$$i_L(\infty) = I_s$$
$$v_R(\infty) = 0.$$

The time constant is $\tau - \frac{L}{R}$. Therefore

$$i_L(t) = I_s(1 - e^{-\frac{L}{R}t})$$
$$v_R(t) = I_s R e^{-\frac{L}{R}t}.$$

---

**Problem 7.4:** For the circuit below, determine the current $i(t)$ "by inspection"




---

**Solution:** At $t = 0$ the capacitor looks like a short circuit. In this case the upper resistors are in parallel and have an equivalent resistance of $2/3\Omega$. The two lower resistors are also in parallel with an effective resistance of $6/5\Omega$. Therefore, the voltage across the $3\Omega$ resistor is (by a voltage divider)

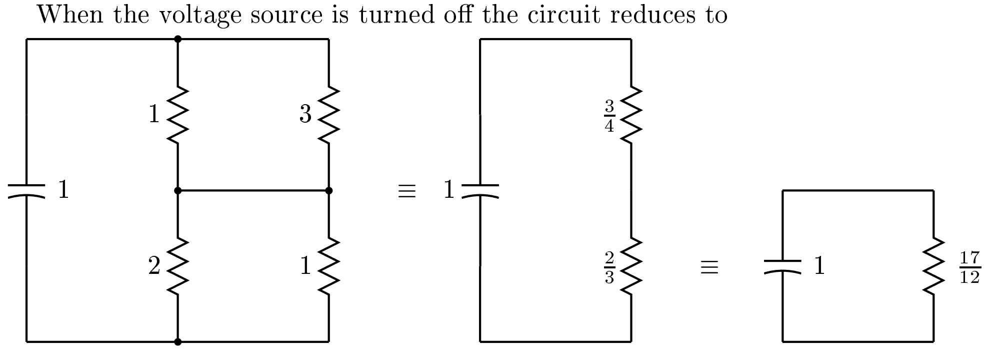
$$\frac{\frac{6}{5}}{\frac{6}{5} + \frac{2}{3}} = \frac{9}{14} \text{ volts.}$$

and $i(0) = \frac{9}{14}\frac{1}{3} = \frac{3}{14}$ volts.

At $t = \infty$ the capacitor looks like an open circuit. By a voltage divider, the voltage across the $3\Omega$ resistor is 3/4 volt and the current is 1/4 amp.

$$i(\infty) = \frac{1}{4} \text{ amp}$$

When the voltage source is turned off the circuit reduces to

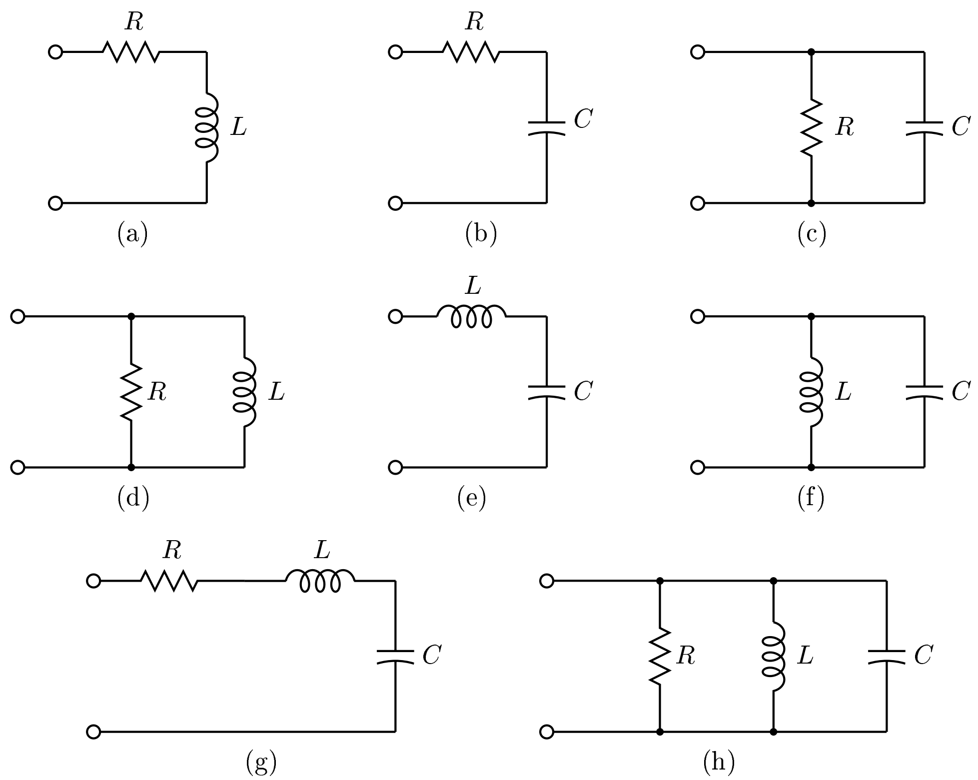


Therefore, $\tau = RC = \frac{17}{12}$ and

$$i(t) = \frac{1}{4} + \left(\frac{3}{14} - \frac{1}{4}\right) e^{-\frac{12}{17}t} = \frac{1}{4} - \frac{1}{28} e^{-\frac{12}{17}t}$$

---

**Problem 7.5:** The *driving-point impedance* of a two-terminal network is the ratio of the Laplace transform of its terminal voltage to the Laplace transform of the current entering the positive terminal. For each of the networks below, determine the driving-point impedance. Express your answers as ratios of polynomials in s.




---

**Solution:**

(a) $Z_a(s) = R + Ls$

(b) $Z_b(s) = R + \frac{1}{CS} = \frac{RCs+1}{Cs}$

(c) $Z_c(s) = \frac{\frac{R}{Cs}}{R+\frac{1}{Cs}} = \frac{R}{RCs+1}$

(d) $Z_d(s) = \frac{RLs}{R+Ls}$

(e) $Z_e(s) = Ls + \frac{1}{Cs} = \frac{LCs^2+1}{Cs}$

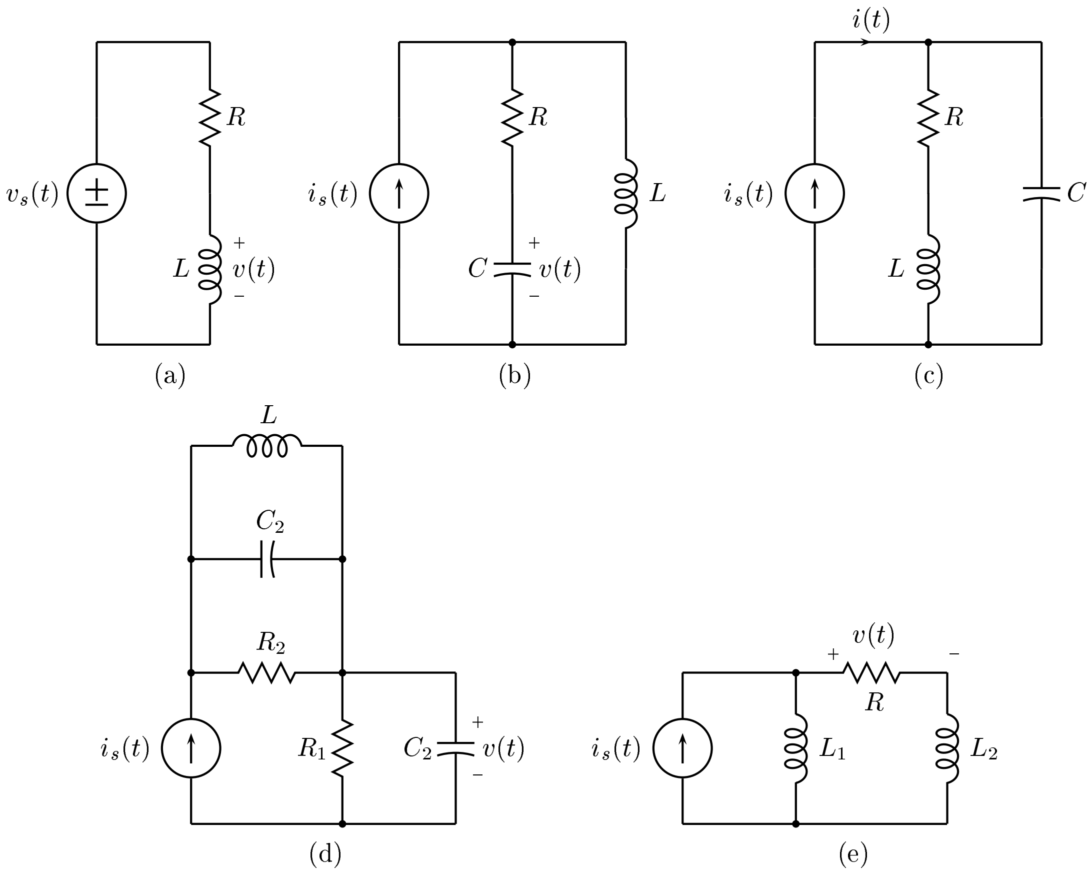
(f) $Z_f(s) = \frac{\frac{L}{C}}{Ls+\frac{1}{Cs}} = \frac{LCs^2+1}{Cs}$

(g) $Z_g(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs}$

(h) $Z_h(s) = \frac{1}{\frac{1}{R}+\frac{1}{Ls}+Cs} = \frac{RLs}{RLCs^2+Ls+R}$

---

**Problem 7.6:** For each of the networks below determine the system function relating the indicated output variable to the excitation (input). Your answers should be expressed as ratios of polynomials in $s$.




---

**Solution:**

(a) $H_a(s) = \frac{Ls}{Ls+r}$

(b) $H_b(s) = \frac{1}{Cs} \cdot \frac{Ls}{R+\frac{1}{Cs}+Ls} = \frac{Ls}{LCs^2+RCs+1}$

(c) $H_c(s) = 1$

(d) $H_d(s) = \frac{1}{C_1 s} \cdot \frac{R_1}{R_1+\frac{1}{C_1 s}} = \frac{R_1}{R_1 C_1 s+1}$

(e) $H_e(s) = \frac{L_1 s}{R+(L_1+L_2)s} \cdot R = \frac{RL_1 s}{(L_1+L_2)s+R}$

---