GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course EE 2250

Electric Circuit Analysis

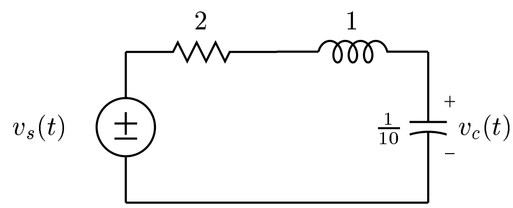
March 4, 1999

Problem Set #8—Solutions

Problem 8.1: (a) For the network below determine the system function.

(b) Find $v_c(t)$ for all time if the network is at rest for $t < 0$ and $v_s(t) = u(t)$.

(c) Find $v_c(t)$ for all time if the network is at rest for $t < 0$ and $v_s(t) = 2\delta(t)$.



Solution:

(a) We can use a voltage divider to determine the system function.

$$H(s) = \frac{V_c(s)}{V_s(s)} = \frac{\frac{10}{s}}{s + 2 + \frac{10}{s}} = \frac{10}{(s+1)^2 + 9}$$

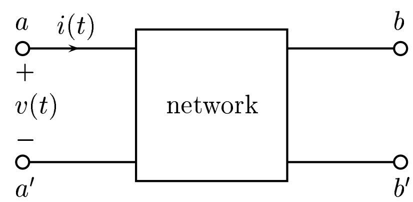
(b)

$$\begin{aligned} V_c(s) &= \frac{10}{(s+1-j3)(s+1+j3)} \\ &= \frac{1}{s} + \frac{-\frac{1}{2} + j\frac{1}{6}}{s+1+j3} + \frac{-\frac{1}{2} - j\frac{1}{6}}{s+1-j3} \\ v_c(t) &= \left(1 - e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t\right) u(t) \end{aligned}$$

(c) Since $2\delta(t) = 2\frac{du(t)}{dt}$, for this input $v_c(t)$ will be twice the derivative of the voltage found in part (b).

$$\begin{aligned} v_c(t) &= 2\frac{d}{dt}\left(1 - e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t\right) u(t) \\ &= 2\left(-3e^{-t}\sin 3t + e^{-t}\cos 3t - e^{-t}\cos 3t + \frac{1}{3}e^{-t}\sin 3t\right) \\ &= -\frac{16}{3}e^{-t}\sin 3t u(t). \end{aligned}$$

Problem 8.2:



For the two-terminal pair circuit shown above the relation between $v(t)$ and $i(t)$ is

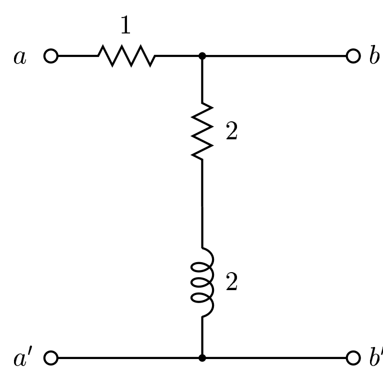
$$v(t) = 3i(t) + 2\frac{di(t)}{dt}$$

when the terminals b and b' are open-circuited and

$$v(t) = i(t)$$

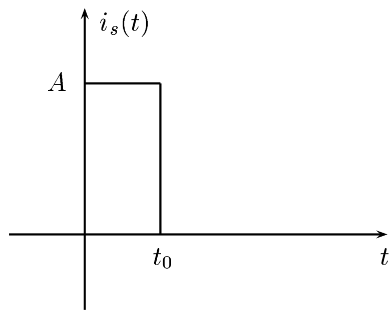
when the terminals b and b' are short-circuited. Determine a possible circuit having these properties.

Solution: The answer to this problem is not unique. Here is one possibility



Problem 8.3: In a particular communication problem, data in the form of a sequence of binary numbers "one" and "zero" are to be coded in such a way that a "one" is represented by four cycles of a sinusoid and a "zero" by the absence of a signal. As one part of the transmitter, a circuit with the following specifications is required:

1. The excitation is a current source $i_s(t)$ that is a pulse of amplitude A starting at $t = 0$ and ending at $t = t_0$ as shown below.



2. The response is a voltage $v(t)$. It is two be zero for $t < 0$ and must be given by $v(t) = B \sin \omega_0 t$ for four complete cycles starting at $t = 0$ and be zero thereafter.

Design a circuit that meets the above specifications using only inductors and capacitors. Determine the width of the input current pulse t_0 and values of the inductors and capacitors in terms of ω_0, A, and B.

Solution: First we see that the input pulse is equivalent to a step that turns on at $t = 0$ followed by a negative step at $t = t_0$. The first step must initiate a sinusoid and the negative step must initiate a sinusoid with the opposite sign to cancel it exactly. Taken together, this means that there must be exactly four cycles in t_0 seconds.

$$\text{period} = \frac{2\pi}{\omega_0} = \frac{t_0}{4}$$

Thus, $t_0 = \frac{8\pi}{\omega_0}$.

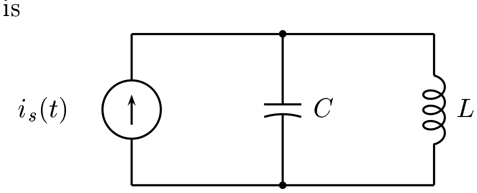
The step response of the circuit is

$$V(s) = \frac{B\omega_0}{s^2 + \omega_0^2} = \frac{A}{s} H(s)$$

which means that for the circuit that we are trying to build we must have

$$H(s) = \frac{s\omega_0 \frac{B}{A}}{s^2 + \omega_0^2}.$$

One possible circuit is



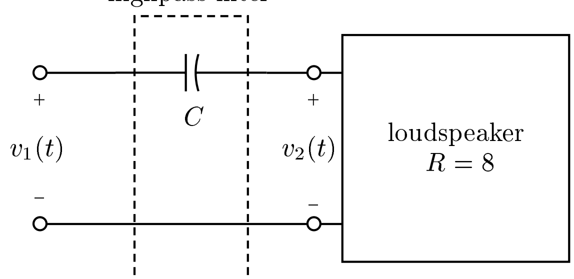
The output voltage is the voltage across any of the elements. For this circuit

$$V(s) = \frac{\frac{Ls}{Cs}}{\frac{1}{Cs} + Ls} = \frac{\frac{1}{C}s}{s^2 + \frac{1}{LC}}$$

Matching terms, this gives

$$C = \frac{B\omega_0}{A}$$
$$L = \frac{A\omega_0}{B}.$$

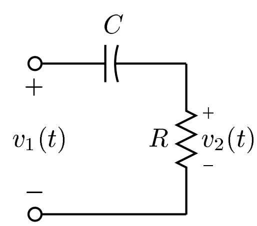
Problem 8.4: Many loudspeaker systems consist of two loudspeakers: the woofer, which reproduces the low frequency part of the signal, and the tweeter, which reproduces the high frequency part of the signal. A crossover network is used to select the high frequency part of the signal and feed it into the tweeter. Such a network functions as a highpass filter. The entire audio signal is applied at the terminals $a - a'$.



(a) Assuming that the equivalent circuit for the tweeter consists of just a resistor with a resistance of R, plot the pole-zero pattern of the system function that relates $v_2(t)$ to $v_1(t)$ and sketch the frequency response curves (magnitude and angle).

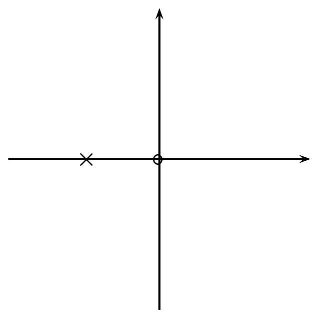
(b) If $R = 8\Omega$, find the value of the capacitance C so that the half-power frequency of the highpass filter is 5 kHz ($= 2\pi(5000)$ rad/s).

Solution:



$$H(s) = \frac{R}{R + \frac{1}{Cs}} = \frac{s}{s + \frac{1}{RC}}$$

(a)



(b)

$$\frac{1}{RC} = 2\pi(5000)$$

Thus,

$$C = \frac{1}{80,000\pi} \approx 4\mu F$$

Problem 8.5: For each of the pole-zero plots below, determine which, if any, of the sketches of magnitude versus frequency in Fig 1 and the sketches of phase versus frequency in Fig 2 could result. You should be able to solve this problem by visualizing the appropriate vectors in the s-plane.

Solution:

(a) The magnitude must be infinite at $\omega = 0$ and the phase must jump from $\pi/2$ to $-\pi/2$ at that frequency and decay to zero as $\omega \to \infty$.

(b) This is called an all-pass filter. The magnitude is constant for all frequencies, since for any point on the $j\omega$ axis, the distance to the zero is the same as the distance to the pole. The phase is π at $\omega = 0$ and decays to zero for large ω since the zero angle will be close to the pole angle.

(c) The double zero at the origin means that the frequency response must be zero at $\omega = 0$ and also that its derivative must be zero. Since there are more poles than zeros, the magnitude will increase linearly with frequency for large frequencies. The phase response must have a discontinuity of size 2π at zero and must arymptotically approach $\pi/2$ because of the excess zero.

(d) This system exhibits resonance, but since there is no zero at zero frequency, the magnitude response does not go to zero at that frequency. The phase must be zero at zero frequency and must approach $-\pi$ for large frequencies with a slope that peaks at $\omega = b$.

(e) The magnitude response must also exhibit a peak here at $\omega = b$, but it must also go to zero at $\omega = 0$ because of the zero. The phase response will have a discontinuity of π at $\omega = 0$ because of the zero at the origin, and the asymptotic value will be $-\pi/2$ because of the excess pole. The slope of the phase curve will be a maximum around $\omega = b$.

(f) There is a peak in the magnitude curve at $\omega = b$ and a minimum at $\omega = b + c$. The values at zero and ∞ will both approach constants. The phase curve must go to zero at both low and high frequencies, but will exhibit at lot of change at frequencies that are close to the poles and zeros.

Problem 8.6: Determine $H(j\omega)$ from the asymptotic Bode plot below.

Solution: By looking at the plot we see that there are zeros at $s = 0$ and $s = -10$ corresponding to the increases in slope, and poles at $s = -0.7$, $s = -100$, and $s = -600$ corresponding to the decreases, assuming that all of the real poles and zeros lie in the left half plane (which cannot be ascertained from the magnitude plot itself). This gives

$$H(s) = \frac{Ks(s + 10)}{(s + 0.7)(s + 100)(s + 600)}$$

To determine K, we note that the gain of the filter at $\omega = 1$ (i.e., $s = j$) is 1, which means $\frac{K \cdot 10}{100 \cdot 600} \approx 1$ or $K = 6000$.

$$H(s) = \frac{6000s(s + 10)}{(s + 0.7)(s + 100)(s + 600)}$$