GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Course EE 2250

Electric Circuit Analysis

March 11, 1999

Problem Set #9—Solutions

Problem 9.1: (Dorf and Svoboda problem 16.3-4) Highpass Butterworth filters have system functions of the form

$$H_{hp}(s) = \frac{\pm k s^n}{D_n(s)}$$

where n is the order of the filter, $D_n(s)$ denotes the denominator polynomial of the corresponding lowpass filter, and k is the passband gain. Obtain the system function of a fourth-order Butterworth highpass filter having a cutoff frequency of 500 Hz. and a passband gain of 5.

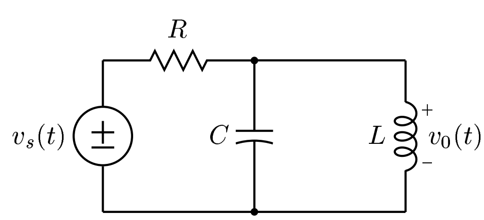
Solution:

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

is a fourth-order Butterworth lowpass filter with a cutoff frequency of 1 rad/s and a gain of 1. Converting this to a highpass with a gain of 5 and a cutoff frequency of $2\pi(500)$ results in the system function

$$\begin{aligned} H_{hp}(s) &= \frac{5\left(\frac{s}{2\pi(500)}\right)^4}{\left[\left(\frac{s}{2\pi(500)}\right)^2 + 0.765\left(\frac{s}{2\pi(500)}\right) + 1\right]\left[\left(\frac{s}{2\pi(500)}\right)^2 + 1.848\left(\frac{s}{2\pi(500)}\right) + 1\right]} \\ &= \frac{5s^4}{[s^2 + 2403.318\,s + 9.870\times 10^6][s^2 + 5805.663\,s + 9.870\times 10^6]} \end{aligned}$$

Problem 9.2: (Dorf and Svoboda problem 16.4.1) The circuit shown below is a second-order bandpass filter. Design this filter to have $k = 1$, $\omega_0 = 1000$ rads/s and $Q = 1$.



Solution:

$$H(s) = \frac{\frac{\frac{L}{C}}{Ls+\frac{1}{Cs}}}{\frac{\frac{L}{C}}{Ls+\frac{1}{Cs}} + R} = \frac{Ls}{RLCs^2 + Ls + R} = \frac{\frac{1}{RC}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

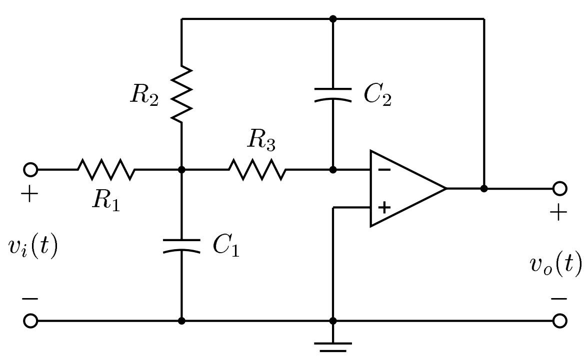
From this we see that

$$\omega_0^2 = \frac{1}{LC} = (1000)^2$$
$$\frac{\omega_0}{Q} = \frac{1}{RC}$$
$$k = 1$$

Therefore, if we let $C = 10\mu F = 10^{-5}F$, we get $L = 0.1H = 100mH$, $R = 100\Omega$.

Problem 9.3: (a) Compute the system function of the circuit below.

(b) Select convenient values of R_1, R_2, R_3, C_1 and C_2 to obtain a lowpass filter with a cutoff frequency of $\omega_0 = 2000$ rad/s and $Q = 8$.



Solution: We can write KCL equations at the opposite nodes of the resistor R_3. Let the potential at the left node be $E(s)$; at the right node it is 0.

$$\frac{E(s) - V_{in}(s)}{R_1} + C_1 sE(s) + \frac{E(s) - V_{out}(s)}{R_2} + \frac{E(s)}{R_3} = 0$$
$$-C_2 sV_{out}(s) - \frac{1}{R_3}E(s) = 0.$$

From the latter equation

$$E(s) = -R_3C_2sV_{out}(s).$$

Substituting

$$\frac{-R_3C_2sV_{out}(s) - V_{in}(s)}{R_1} - R_3C_1C_2s^2V_{out}(s) - \frac{R_3C_2s}{R_2}V_{out}(s) - \frac{1}{R_2} - C_2sV_{out}(s) = 0$$

Solving

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{\frac{1}{R_1 R_3 C_1 C_2}}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_3 C_1}\right) s + \frac{1}{R_2 R_3 C_1 C_2}}$$

We want

$$\frac{1}{R_2 R_3 C_1 C_2} = (2000)^2$$
$$\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_3 C_1} = \frac{2000}{8} = 250.$$

One possible solution is to let $C_1 = 0.1\mu F$, $C_2 = 0.001\mu F$, $R_2 = R_3 = 50k\Omega$, and $R_1 = 2800\Omega$.

Problem 9.4: Work Problem 16.4-6 in the text by Dorf and Svoboda.

Solution: Write KCL equations at the inputs to the two opamps. The potential at the left is $E(s)$ and at the right it is $V_{out}(s)$.

$$C_1 s[E(s) - V_{in}(s)] + \frac{1}{R_1}[E(s) - V_{out}(s)] = 0$$
$$\frac{1}{R_2} V_{out}(s) + C_2 s[V_{out}(s) - E(s)] = 0$$

We can solve for $H(s) = V_{out}(s)/V_{in}(s)$.

$$H(s) = \frac{s^2}{s^2 + \frac{1}{R_2 C_2} s + \frac{1}{R_1 R_2 C_1 C_2}}$$

From this we want

$$\frac{1}{R_1 C_1} = \frac{1}{R_2 C_2} = 1000$$

One choice is

$$R_1 = R_2 = 10k\Omega; \;\; C_1 = C_2 = 0.1\mu F.$$

Problem 9.5: Work Problem 16.4-8 in the text by Dorf and Svoboda.

Solution: If all of the impedances are scaled up by a factor of 1000, the capacitor values will fall within the acceptable range. This makes the 10Ω resistor into a $10k\Omega$ resistor, the 20Ω resistor becomes a $20\text{k}\Omega$ resistor, the $100\mu\text{F}$ capacitor becomes a 0.1μF capacitor, and the 500μF capacitor becomes a 0.5μF capacitor. The system function before and after the transformation can be found from the inverting amplifier configuration

$$H(s) = -\frac{\frac{\frac{R_2}{C_2 s}}{R_2 + \frac{1}{C_2 s}}}{R_1 + \frac{1}{C_1 s}} = -\frac{100}{(s+200)(s+500)}.$$

Problem 9.6: Work Problem 16.5-9 in the text by Dorf and Svoboda.

Solution:

(a) This opamp is also configured as an inverting amplifier. Therefore, for each stage

$$H_1(s) = -\frac{\frac{\frac{R_2}{Cs}}{R_2 + \frac{1}{Cs}}}{R_1} = \frac{R_2/R_1}{1 + R_2 Cs}$$

To cut off at 1000 rad/s, we want

$$R_2 C = 0.001$$

and for a gain of 1 at $s = 0$ we want $R_1 = R_2$. Therefore, we choose

$$C = 0.1\mu F; \;\; R_1 = R_2 = 10^4\Omega = 10k\Omega$$

(b) For each stage for frequencies above 1000 rad/s, the response rolls off at 20 dB/decade. Therefore, the gain for both stages together at $\omega = 10,000$ rad/s is -40 dB.
