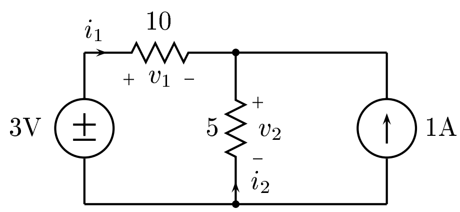
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE 2250
Electric Circuit Analysis

Quiz #1—Solutions

Monday, January 25, 1999

Problem Q1.1: In the circuit below both source waveforms (and all of the element variables) are constant. Compute the values of i_1, v_1, i_2, and v_2.



Solution: The basic network has two elements, two nodes, and one mesh. Thus, we need to write two element relations, one KCL equation, and one KVL equation in order to solve for the four element variables. These equations are:

$$\begin{aligned} R_1: &\quad v_1 = 10 i_1 \\ R_2: &\quad v_2 = -5 i_2 \quad \text{Notice the reference direction.} \\ \text{KCL}: &\quad i_1 + i_2 = -1 \\ \text{KVL}: &\quad v_1 + v_2 = 3. \end{aligned}$$

Substituting the element relations into the KVL equation gives:

$$\begin{aligned} \text{KVL}: &\quad 10 i_1 - 5 i_2 = 3 \\ 5\times\text{KCL}: &\quad 5 i_1 + 5 i_2 = -5. \end{aligned}$$

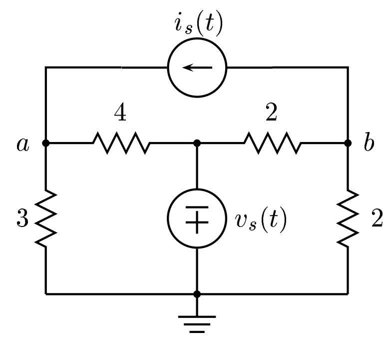
Adding these two equations gives

$$15 i_1 = -2.$$

Knowing this current, we can determine all of the remaining element variables

$$\begin{aligned} i_1 &= -\frac{2}{15} A \\ v_1 &= -\frac{4}{3} V \\ i_2 &= -\frac{13}{15} A \\ v_2 &= \frac{13}{3} V \end{aligned}$$

Problem Q1.2:



We propose to solve the above circuit using the node method.

(a) Write the KCL equations at nodes a and b in terms of the node potentials at those nodes, $e_a(t)$ and $e_b(t)$.

(b) Put your equations in matrix-vector form by supplying the missing constants in the framework below.

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} i_s(t) + \begin{bmatrix} \quad \\ \quad \end{bmatrix} v_s(t)$$

(c) Solve them for $e_a(t)$ and $e_b(t)$.

Solution:

(a) Observe that the node potential of the node in the center of the circuit is $-v_s(t)$. With a potential defined at each node of the circuit the equations are particularly simple to write.

$$\frac{1}{3}e_a(t) + \frac{1}{4}[e_a(t) + v_s(t)] = i_s(t)$$
$$\frac{1}{2}e_b(t) + \frac{1}{2}[e_b(t) + v_s(t)] = -i_s(t)$$

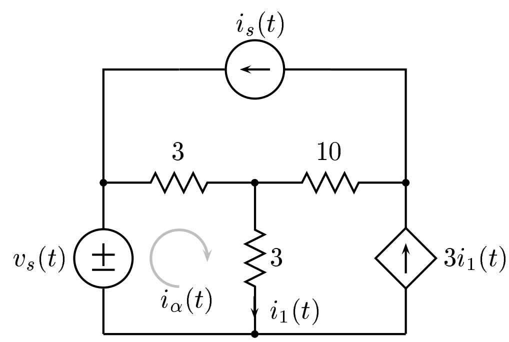
(b)

$$\begin{bmatrix} \frac{7}{12} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_b(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} i_s(t) + \begin{bmatrix} -\frac{1}{4} \\ -\frac{1}{2} \end{bmatrix} v_s(t)$$

(c) These equations are particularly simply to solve

$$e_a(t) = \frac{12}{7}i_s(t) - \frac{3}{7}v_s(t)$$
$$e_b(t) = -i_s(t) - \frac{1}{2}v_s(t)$$

Problem Q1.3:



(a) How many meshes are present in the basic network?

(b) How many nodes are present in the basic network?

(c) Write a KVL equation over the indicated mesh of the complete network. Express your answer in terms of the mesh current $i_\alpha(t)$, $i_1(t)$, $v_s(t)$, and $i_s(t)$.

(d) Express $i_1(t)$ as a function of $i_\alpha(t)$.

(e) Determine $i_1(t)$ as a function of $i_s(t)$ and $v_s(t)$.

Solution:

(a) In the basic network both the independent and dependent current sources become open circuits and the voltage source becomes a short circuit. This leave a single mesh in the basic network.

(b) The basic network contains two nodes—the one in the center of the circuit and the one at the bottom.

(c) $-v_s(t) + 3[i_\alpha(t) + i_s(t)] + 3[i_\alpha(t) + 3i_1(t)] = 0$

(d) Since $i_1(t)$ is the current flowing down through the vertical 3Ω resistor,

$$i_1(t) = i_\alpha(t) + 3i_1(t)$$

from which it follows that

$$i_1(t) = -\frac{1}{2}i_\alpha(t).$$

(e) We can substitute the result from part (d) into the equation from part (c). This gives the "new" KVL equation

$$3i_\alpha(t) - \frac{3}{2}i_\alpha(t) = v_s(t) - 3i_s(t)$$

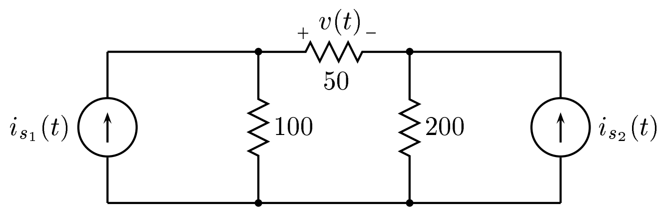
from which

$$i_\alpha(t) = \frac{2}{3}v_s(t) - 2i_s(t)$$

and

$$i_1(t) = -\frac{1}{3}v_s(t) + i_s(t)$$

Problem Q1.4:



(a) Which method, the mesh method or the node method, will result in fewer equations to solve in order to determine $v(t)$?

(b) Determine $v(t)$ using the method that you selected in (a).

Solution:

(a) The mesh method requires writing only one KVL equation, because the basic network contains only one mesh. The node method, on the other hand requires writing two KCL equations because the basic network contains three nodes. Therefore, the mesh method will result in fewer equations.

(b) Let $i_{s_1}(t)$ be a clockwise mesh current in the left node of the complete circuit, $i(t)$ be a clockwise mesh current in the center mesh (the only one around which we write a KVL equation), and $-i_{s_2}(t)$ be the clockwise mesh current around the right mesh. Then the KVL equation is

$$100[i(t) - i_{s_1}(t)] + 50i(t) + 200[i(t) + i_{s_2}(t)] = 0.$$

Solving for $i(t)$ gives

$$350i(t) = 100i_{s_1} - 200i_{s_2}(t)$$
$$i(t) = \frac{2}{7}i_{s_1} - \frac{4}{7}i_{s_2}(t)$$

From this

$$v(t) = \frac{100}{7}i_{s_1}(t) - \frac{200}{7}i_{s_2}(t).$$
