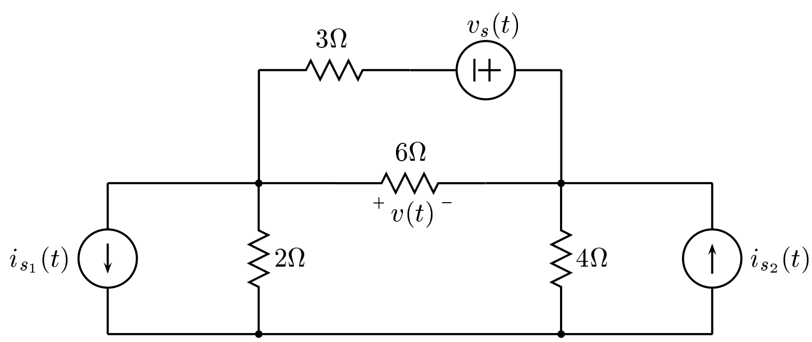
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE 2250
Electric Circuit Analysis

Quiz #2 – Solutions

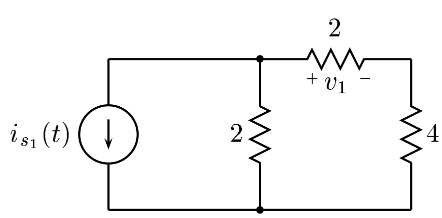
Problem Q2.1: Find $v(t)$ in the circuit below in terms of $v_s(t)$, $i_{s_1}(t)$, and $i_{s_1}(t)$.



Solution: The reason that this problem is on the quiz, although it can be solved using methods tested in Quiz #1, is to stress the utility of superposition, series/parallel combination rules, and voltage/current dividers. Using superposition

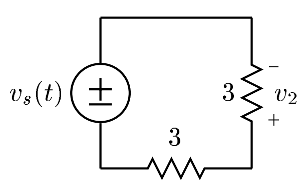
$$v(t) = v_1(t) + v_2(t) + v_3(t),$$

where the component voltages can be found from the following circuits:



Clearly,

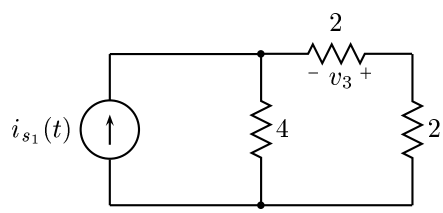
$$v_1(t) = -2 \cdot \frac{2}{2+2+4} i_{s_1}(t) = -\frac{1}{2} i_{s_1}(t).$$



Here,

$$v_2(t) = -\frac{1}{2} v_s(t).$$

Finally,



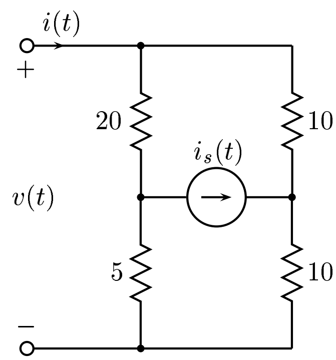
Here,

$$v_3(t) = -2\left(\frac{1}{2}\right) i_{s_2}(t).$$

Therefore,

$$v(t) = -\left[\frac{1}{2}i_{s_1}(t) + \frac{1}{2}v_s(t) + i_{s_2}(t)\right].$$

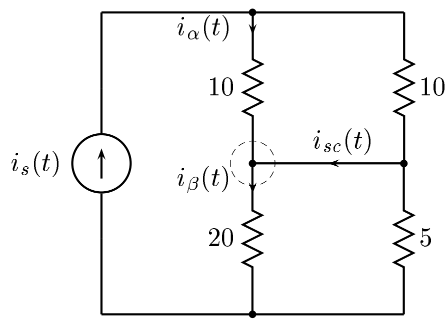
Problem Q2.2: Find the Norton equivalent of the two-terminal network shown below.



Solution: To find the Norton (or Thévenin) equivalent circuit it is sufficient to measure the open-circuit voltage $v_{oc}(t)$ and the short-circuit current $i_{sc}(t)$. To measure $v_{oc}(t)$ we leave the terminals open and observe by a current divider that $\frac{1}{3}i_s(t)$ will flow upward through the loop containing the 10Ω and 20Ω resistor and that $\frac{2}{3}i_s(t)$ will flow through the lower loop. Therefore, relative to the node connecting the two 10Ω resistors, the potential at the + terminal is $-(10/3)i_s(t)$ and the potential at the − terminal is $-(20/3)i_s(t)$. Therefore,

$$v_{oc}(t) = -\frac{10}{3}i_s(t) - \left[-\frac{20}{3}i_s(t)\right] = \frac{10}{3}i_s(t).$$

To measure $i_{sc}(t)$ it helps to redraw the circuit.



By a current divider at the upper pair of resistors

$$i_\alpha(t) = \frac{1}{2} i_s(t)$$

and by one at the lower pair

$$i_\beta(t) = \frac{1}{5} i_s(t).$$

Applying KCL at the circled node

$$i_\beta(t) - i_\alpha(t) = i_{sc}(t)$$

from which we see that

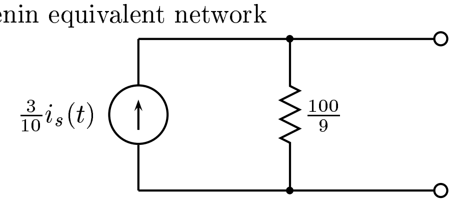
$$i_{sc}(t) = -\frac{3}{10} i_s(t).$$

Therefore,

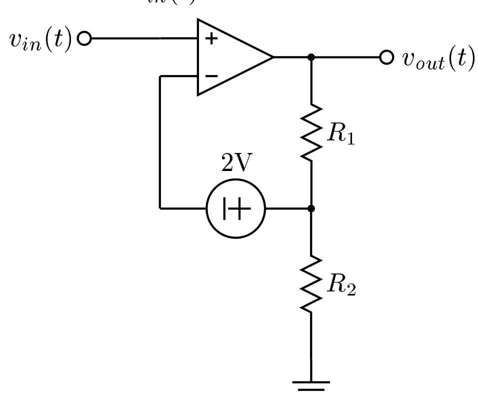
$$R_N = -\frac{v_{oc}(t)}{i_{sc}(t)} = \frac{100}{9}$$

$$i_N(t) = -i_{sc}(t),$$

which gives the Thévenin equivalent network



Problem Q2.3: Express $v_{out}(t)$ in terms of $v_{in}(t)$.



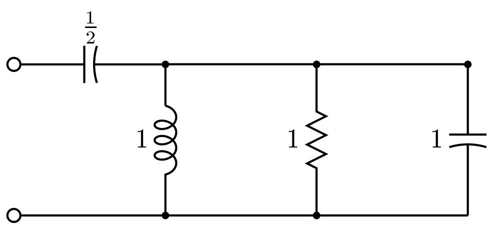
Solution: Applying KCL at the node where the resistors are connected.

$$\frac{v_{in}(t)+2}{R_2}+\frac{v_{in}(t)+2-v_{out}(t)}{R_1}=0.$$

Solving for $v_{out}(t)$ gives

$$v_{out}(t)=\left(1+\frac{R_1}{R_2}\right)(v_{in}+2)$$

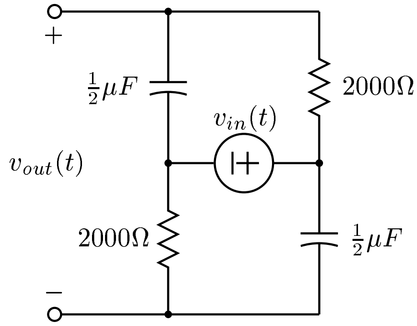
Problem Q2.4: Find the equivalent impedance of the two-terminal network drawn below.



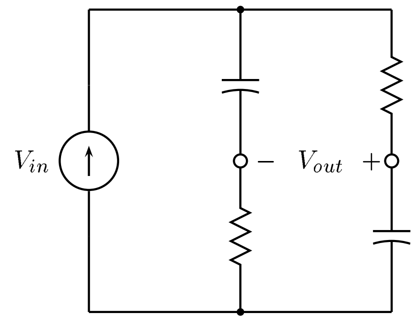
Solution:

$$\begin{aligned}Z_{eq}(j\omega) &= \frac{2}{j\omega}+\frac{1}{\frac{1}{j\omega}+j\omega+1}=\frac{2}{j\omega}+\frac{j\omega}{1-\omega^2+j\omega}\\ &= \frac{2(1-\omega^2+j\omega)+(j\omega)^2}{(j\omega)(1-\omega^2+j\omega)}\\ &= \frac{2-3\omega^2+j2\omega}{-\omega^2+j\omega(1-\omega^2)}\end{aligned}$$

Problem Q2.5: Find $v_{out}(t)$ if $v_{in}(t)=3+4\sin(1000t)$. The terminals are *open-circuited.*



Solution: We can redraw this circuit as



Then

$$\begin{aligned} V_{out} &= \left(\frac{\frac{1}{jC\omega}}{\frac{1}{jC\omega}+R} - \frac{R}{\frac{1}{jC\omega}+R} \right) V_{in} \\ &= \frac{1-jRC\omega}{1+jRC\omega} V_{in} \end{aligned}$$

Now, we observe

$$\begin{aligned} \omega = 0: &\quad V_{out} = V_{in} \\ \omega = 1000: &\quad V_{out} = \frac{1-j}{1+j} V_{in} = e^{-j\frac{\pi}{2}} V_{in} \end{aligned}$$

Therefore,

$$v_{out}(t) = 3 + 4\sin(1000t - \frac{\pi}{2}) = 3 - 4\cos(1000t).$$