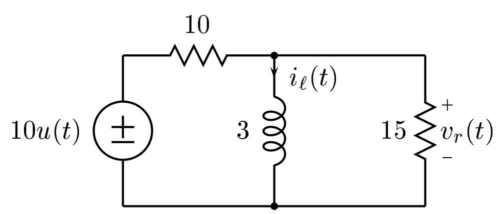
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE 2250
Electric Circuit Analysis

Quiz #3—Solutions

Thursday, March 4, 1999

Problem Q3.1: This problem concerns the following circuit.



(a) Find $v_r(t)$ for $t > 0$ if $i_\ell(0) = 0$.

(b) Find $v_r(t)$ for $t > 0$ if $i_\ell(0) = 5$.

Solution:

(a) At $t = 0$ the inductor looks like an open circuit.

$$v_r(0) = \frac{15}{25} \cdot 10 = 6$$

At $t = \infty$ the inductor looks like a short circuit.

$$v_r(\infty) = 0.$$

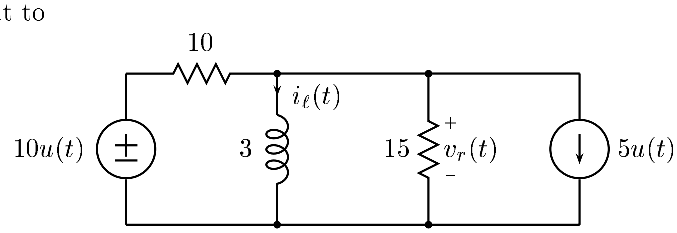
We turn off the source to determine the time constant

$$\tau = \frac{L}{R_{eq}} = \frac{3}{6} = \frac{1}{2}.$$

Therefore,

$$v_r(t) = 6e^{-2t} \quad t \geq 0.$$

(b) We can incorporate a current source to handle the initial condition. This changes the circuit to



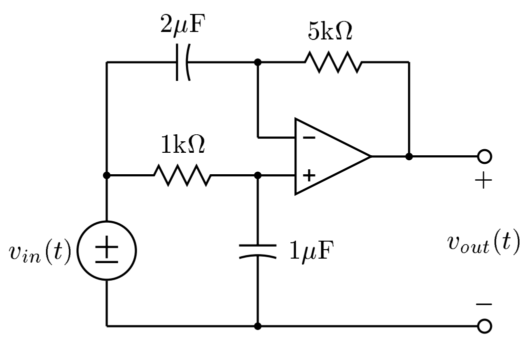
The time constant is the same as before. The value at $t = \infty$ is also the same as before. At $t = 0$

$$v_r(0) = 6 - 30 = -24.$$

Therefore,

$$v_r(t) = -24e^{-2t} \quad t \geq 0.$$

Problem Q3.2:



Find the system function $H(s) = V_{out}(s)/V_{in}(s)$ for the above circuit.

Solution: Let the node potentials at the two nodes connected to the inputs of the opamp be $E(s)$. (They are the same.) Then we get for KCL equations

$$C_1 s[E(s) - V_{in}(s)] + \frac{1}{R_2}[E(s) - V_{out}(s)] = 0$$
$$\frac{1}{R_1}[E(s) - V_{in}(s)] + C_2 s E(s) = 0$$

From the second equation

$$E(s) = \frac{1}{(1 + R_1 C_2 s)} V_{in}(s).$$

From the first

$$[1 + C_1 R_2 s]E(s) - R_2 C_1 s V_{in}(s) = V_{out}(s)$$
$$\left[\frac{1 + C_1 R_2 s}{1 + C_2 R_1 s} - R_2 C_1 s\right] V_{in}(s) = V_{out}(s)$$

From this

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1 - C_1 C_2 R_1 R_2 s^2}{1 + C_2 R_1 s} = \frac{1 - 10^{-5} s^2}{1 + 10^{-3} s}$$

Problem Q3.3: Find the signal $x(t)$ for $t > 0$ if its Laplace transform is

$$X(s) = \frac{s^3}{(s+1)([s+1]^2+4)}.$$

Solution:

$$X(s) = \frac{s^3}{(s+1)(s^2+2s+5)} = 1 + \frac{A}{s+1} + \frac{B}{s+1-j2} + \frac{B^*}{s+1+j2}$$

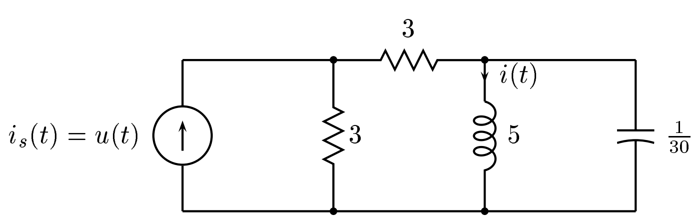
$$A = \lim_{s\to -1} \frac{s^3}{s^2+2s+5} = -\frac{1}{4}$$

$$B = \lim_{s\to -1+j2} \frac{s^3}{(s+1)(s+1+j2)} = -\frac{11}{8} - \frac{1/2}{j2}$$

Therefore,

$$\begin{aligned} x(t) &= \delta(t) - \frac{1}{4}e^{-t} + \left(-\frac{11}{8} + \frac{1/2}{j2}\right) e^{-t}e^{j2t} + \left(-\frac{11}{8} - \frac{1/2}{j2}\right) e^{-t}e^{-j2t} \\ &= \delta(t) - e^{-t}\left(\frac{1}{4} + \frac{11}{4}\cos 2t - \frac{1}{2}\sin 2t\right), \quad t \geq 0 \end{aligned}$$

Problem Q3.4:



(a) For the circuit above, determine the system function $H(s)$ that relates the output $i(t)$ to the input $i_s(t)$, i.e. find

$$H(s) = \frac{I(s)}{I_s(s)}$$

(b) Determine $i(t)$ for all values of t if $i_s(t) = u(t)$. Assume that the circuit is at initial rest for $t < 0$.

Solution:

(a)

$$H(s) = \frac{3}{6 + \frac{5s\cdot\frac{30}{s}}{5s+\frac{30}{s}}} \cdot \frac{\frac{30}{s}}{5s+\frac{30}{s}} = \frac{3\cdot\frac{30}{s}}{30s + \frac{180}{s} + 150} = \frac{3}{s^2+5s+6}$$

(b)

$$I(s) = \frac{1}{s} \cdot \frac{3}{s^2 + 5s + 6} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = \lim_{s \to 0} \frac{3}{(s+3)(s+2)} = \frac{1}{2}$$
$$B = \lim_{s \to -2} \frac{3}{s(s+3)} = -\frac{3}{2}$$
$$C = \lim_{s \to -3} \frac{3}{s(s+2)} = 1$$

Therefore,

$$i(t) = \left(\frac{1}{2} - \frac{3}{2}e^{-2t} + e^{-3t}\right) u(t)$$
