Georgia Institute of Technology

CSIP
Center for Signal & Image Processing

Streaming Measurements in Compressive Sensing (CS) and L^1 Decoding / Filtering

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Motivation – Measurement update

- A simple problem (which can get complicated)
  - Solve a system of linear equations $$Ax = y$$
  - Solve using least squares minimize $\|A\tilde{x} - y\|_2$
  - Analytical solution exists $$x_0 = (A^T A)^{-1} A^T y$$
- One or more new measurements

$$\begin{bmatrix} A \\ B \end{bmatrix} x = \begin{bmatrix} y \\ w \end{bmatrix} \longleftarrow \text{update}$$

  - Solve a new system of equations ?

$$x_1 = (A^T A + B^T B)^{-1}(A^T y + B^T w)$$

- Least squares → Recursive least squares (RLS)
  - Add new measurements
  - Instead of solving from scratch, just update the already computed solution
    - Easy: small rank update

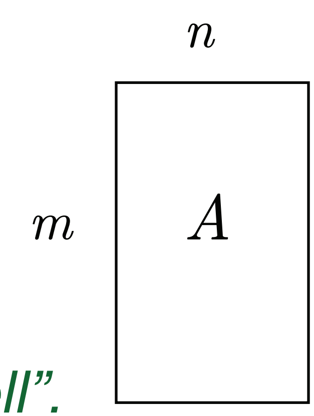
$$\boxed{x_1 = x_0 + (I + B(A^T A)^{-1} B^T)^{-1} (A^T A)^{-1} B^T (w - Bx_0)}$$

L^1 problems

- L^1 norm minimization problems: recover sparse signals from their linear measurements.
- System model: $y = Ax + e$
- L1 Decoding [Candes,Tao]:

  minimize $\|A\tilde{x} - y\|_1$

  "This recovery procedure works unreasonably well".



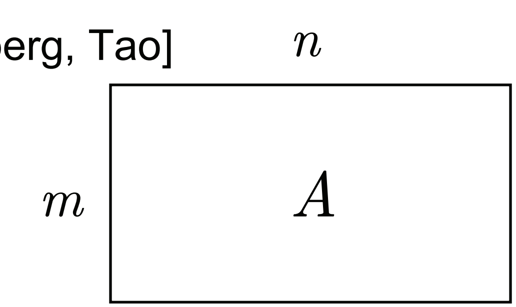
Over-determined system

- Compressed sensing / Compressive sensing
  - Basis pursuit ($e=0$) [Donoho et. al], [Candes, Romberg, Tao]

    minimize $\|\tilde{x}\|_1$ subject to $A\tilde{x} = y$
  - Quadratic constrained [Candes, Romberg, Tao]

    minimize $\|\tilde{x}\|_1$ subject to $\|A\tilde{x} - y\|_2 \leq \sigma$
  - Dantzig selector [Candes,Tao]

    minimize $\|\tilde{x}\|_1$ subject to $\|A^T(A\tilde{x} - y)\|_\infty \leq \tau$



Under-determined system

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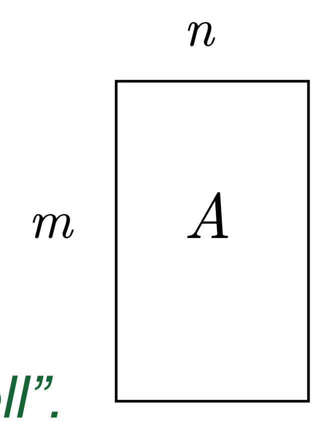
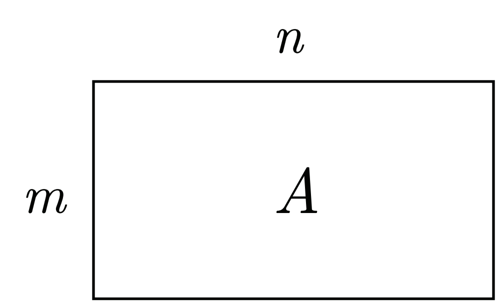


Over-determined system

- Compressed sensing / Compressive sensing
  - Basis pursuit Denoising (BPDN) [Donoho et. al.]
  - Equivalent to Lasso [Tibshirani]

$$\text{minimize} \quad \tau\|\tilde{x}\|_1 + \tfrac{1}{2}\|A\tilde{x} - y\|_2^2$$

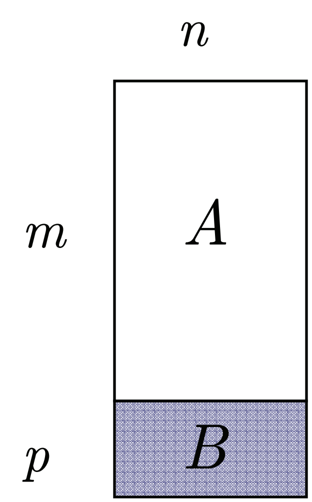


Under-determined system

Measurement update - L^1 problems

- Can we do something similar to RLS in L^1 decoding?
- System model: $$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} x + \begin{bmatrix} e \\ d \end{bmatrix}$$
- L1 Decoding → L1 Filtering?

minimize $\|A\tilde{x} - y\|_1 + \|B\tilde{x} - w\|_1$



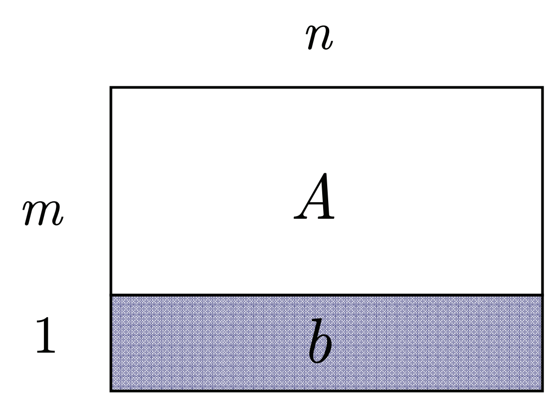
Updated system with p new measurements/codeword

- First: Gross (sparse) errors only!
- Later: If we also have small noise in all the measurements
  - Robust error correction [Candes, Randall]

Measurement update - L^1 problems

- Can we do something similar to RLS in Lasso (or Dantzig selector)?
- System model: $$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} A \\ b \end{bmatrix} x + \begin{bmatrix} e \\ d \end{bmatrix}$$
- Lasso → Dynamic Lasso:

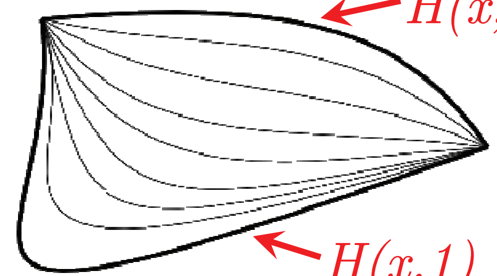
minimize $\quad \tau\|\tilde{x}\|_1 + \frac{1}{2}\|A\tilde{x} - y\|_2^2 + \frac{1}{2}\|b\tilde{x} - w\|_2^2$



Updated system with 1 new measurement

- L1 problems – Dynamic update
  - Not as smooth as least square update
  - Solution can change drastically even with one new measurement
  - Need to move slowly → *iterative method* → *HOMOTOPY*
    - Still easy: each step involves small rank update

Homotopy



- A continuous transformation from one function to another.
- *Say H is a homotopy map between two functions f and g from a space X to Y.*

$H : X \times [0,1] \mapsto Y$ such that $H(x,0) = f(x)$ and $H(x,1) = g(x)$.

- Lasso and LARS homotopy [Osborne et. al.], [Efron et. al.], [Fuchs]
  - Start from an x_0, treat τ as the homotopy parameter and reduce τ towards the desired value.
  - How to track the homotopy path?
    - Optimality conditions!
- Homotopy for Dantzig selector similar in principle [James et. al.], [A., Romberg]

$$\underset{\tilde{x}}{\text{minimize}} \quad \tau\|\tilde{x}\|_1 + \frac{1}{2}\|y - A\tilde{x}\|_2^2$$

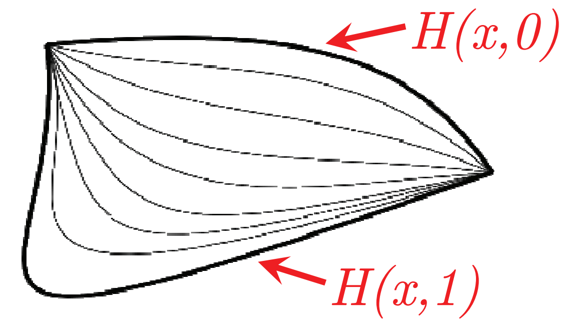
Optimality conditions

(L1.) $A_\Gamma^T(Ax^* - y) = -\tau z$

(L2.) $\|A_{\Gamma^c}^T(Ax^* - y)\|_\infty < \tau$

Iteratively traverse the homotopy path while keeping track of the optimality and feasibility conditions!

Homotopy



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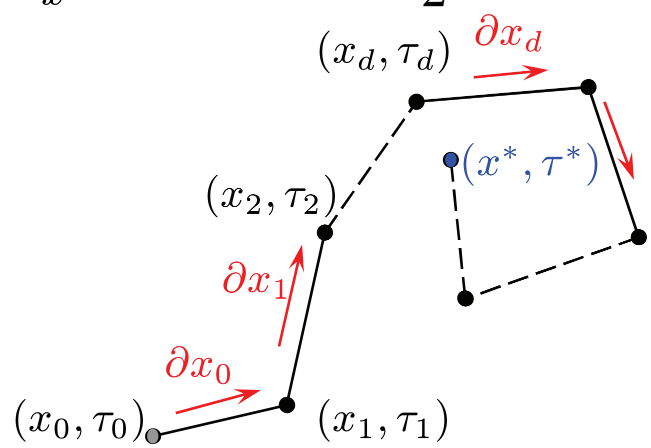


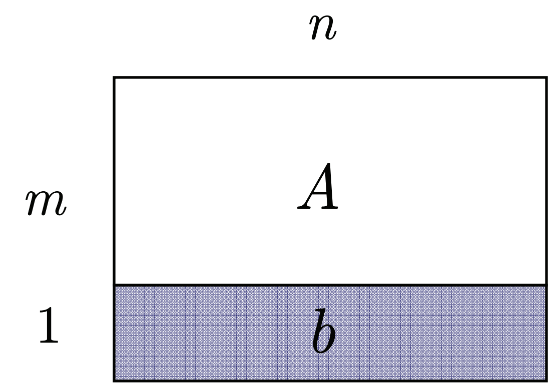
- Homotopy for Dantzig selector similar in principle [James et. al.], [A., Romberg]

Iteratively traverse the homotopy path while keeping track of the optimality and feasibility conditions!

Measurement update - L^1 problems

- Dynamic Lasso:

$$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} A \\ b \end{bmatrix} x + \begin{bmatrix} e \\ d \end{bmatrix}$$



Updated system with 1 new measurement

minimize $\quad \tau\|\tilde{x}\|_1 + \frac{1}{2}\|A\tilde{x} - y\|_2^2 + \frac{1}{2}\|b\tilde{x} - w\|_2^2$

minimize $\quad \tau\|\tilde{x}\|_1 + \frac{1}{2}(\|A\tilde{x} - y\|_2^2 + \epsilon\,\|b\tilde{x} - w\|_2^2), \qquad (L_\epsilon)$

Homotopy parameter: change it from 0 to 1.

How to change ϵ ? (direction, step size) → Optimality conditions!

$$\|A^T(Ax^{(\epsilon)} - y) + \epsilon b^T(bx^{(\epsilon)} - w)\|_\infty \leq \tau, \qquad (L_{\mathrm{opt}})$$

(M1.) $\quad A_\Gamma^T(Ax^{(\epsilon)} - y) + \epsilon b_\Gamma^T(bx^{(\epsilon)} - w) = -\tau z_\epsilon$

(M2.) $\quad \|A_{\Gamma^c}^T(Ax^{(\epsilon)} - y) + \epsilon b_{\Gamma^c}^T(bx^{(\epsilon)} - w)\|_\infty < \tau$

Homotopy update direction - CS

- $x^{(\epsilon_0)}$ is a solution we have for (L_ϵ) at some $\epsilon_0 \in [0,1]$ with support Γ and sign sequence z.
- The update direction ∂x w.r.t. increasing ϵ can be derived using (M1).

$$A_\Gamma^T(A_\Gamma x_\Gamma^{(\epsilon_0)} - y) + A_\Gamma^T A_\Gamma \hat{\partial x} + \epsilon b_\Gamma^T(b_\Gamma x_\Gamma^{(\epsilon_0)} - w) + \epsilon b_\Gamma^T b_\Gamma \hat{\partial x} = -\tau z$$

$$\vdots$$

$$\hat{\partial x} = -(\epsilon - \epsilon_0)(A_\Gamma^T A_\Gamma + \epsilon b_\Gamma^T b_\Gamma)^{-1} b_\Gamma^T (b_\Gamma x_\Gamma^{(\epsilon_0)} - w)$$

$U := A_\Gamma^T A_\Gamma + \epsilon_0 b_\Gamma^T b_\Gamma$

$$\vdots$$

$$\hat{\partial x} = -(\epsilon - \epsilon_0) U^{-1} b_\Gamma^T \left(1 + (\epsilon - \epsilon_0) \underbrace{b_\Gamma U^{-1} b_\Gamma^T}_{u}\right)^{-1} (b_\Gamma x_\Gamma^{(\epsilon_0)} - w).$$

$$\theta := \frac{\epsilon - \epsilon_0}{1 + (\epsilon - \epsilon_0) u}$$

$$\boxed{\partial x = \begin{cases} U^{-1} b_\Gamma^T (w - b x^{(\epsilon_0)}) & \text{on } \Gamma \\ 0 & \text{elsewhere.} \end{cases}}$$

Homotopy update direction - CS

RLS

One homotopy step

$$\boxed{x_1 = x_0 + \frac{(A^TA)^{-1}b^T(w - bx_0)}{1 + b(A^TA)^{-1}b^T}}$$

L1 update

Multiple homotopy steps

$$\partial x = \begin{cases} U^{-1}b_\Gamma^T(w - bx^{(\epsilon_0)}) & \text{on } \Gamma \\ 0 & \text{elsewhere.} \end{cases}$$

$$U = (A_\Gamma^T A_\Gamma + \epsilon_0 b_\Gamma^T b_\Gamma)$$

$$u = b_\Gamma U^{-1} b_\Gamma^T$$

$$\boxed{x^{(\epsilon)} = x^{(\epsilon_0)} + \frac{(\epsilon - \epsilon_0)}{1 + (\epsilon - \epsilon_0)u} U^{-1}b_\Gamma^T(w - bx^{(\epsilon_0)}) \quad \text{on } \Gamma.}$$

Homotopy step size - CS

- Smallest step size such that either one new element enters the support of x or an existing element becomes zero
- But what step size?

$$\|A^T(A(x^{(\epsilon_0)}+\theta\partial x)-y)+\epsilon b^T(b(x^{(\epsilon_0)}+\theta\partial x)-w)\|_\infty \le \tau,$$

$$\|\underbrace{A^T(Ax^{(\epsilon_0)}-y)+\epsilon_0 b^T(bx^{(\epsilon_0)}-w)}_{p_k}+(A^TA+\epsilon b^Tb)\partial x+(\epsilon-\epsilon_0)(bx^{(\epsilon_0)}-w)\|_\infty \le \tau,$$

$$\|p_k+\underbrace{\frac{\epsilon-\epsilon_0}{1+(\epsilon-\epsilon_0)u}}_{\theta}\underbrace{[-(A^TA_\Gamma+\epsilon_0 b^Tb_\Gamma)U^{-1}b_\Gamma^T(bx^{(\epsilon_0)}-w)+b^T(bx^{(\epsilon_0)}-w)]}_{d_k}\|_\infty \le \tau$$

$$\boxed{\|p_k+\theta d_k\|_\infty \le \tau,}$$

where $u=b_\Gamma U^{-1}b_\Gamma^T$ and new $\epsilon=\epsilon_0+\dfrac{\theta}{1-\theta u}$.

Homotopy update - CS

Select step size θ as described below, so either one new element enters the support of x at index (j^+) or an existing element at index (j^-) shrinks to zero. Update support and sign accordingly.

$$|p_k(j) + \theta d_k(j))| = \tau \qquad \text{for all } j \in \Gamma$$

$$|p_k(j) + \theta d_k(j))| \leq \tau \qquad \text{for all } j \in \Gamma^c$$

$$\left.\begin{aligned} \theta^+ &= \min_{j\in\Gamma^c} \left(\frac{\tau - p_k(j)}{d_k(j)}, \frac{\tau + p_k(j)}{-d_k(j)} \right)_+ \\ j^+ &= \operatorname{arg\,min}_{j\in\Gamma^c} \left(\frac{\tau - p_k(j)}{d_k(j)}, \frac{\tau + p_k(j)}{-d_k(j)} \right)_+ \end{aligned}\right\}$$

A new element enters the support

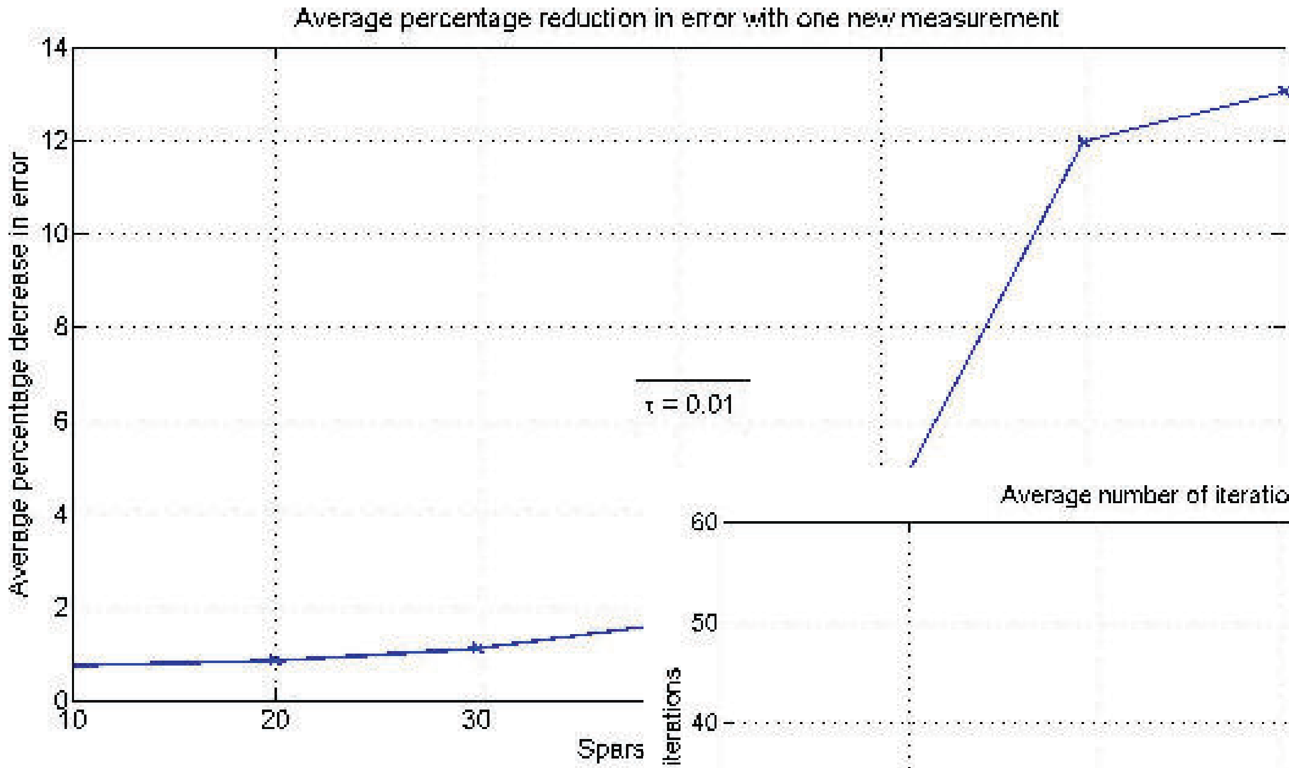
$$\left.\begin{aligned} \theta^- &= \min_{j\in\Gamma} \left(\frac{-x^{(\epsilon_0)}(j)}{\partial x(j)} \right)_+ \\ j^- &= \operatorname{arg\,min}_{j\in\Gamma} \left(\frac{-x^{(\epsilon_0)}(j)}{\partial x(j)} \right)_+ \end{aligned}\right\}$$

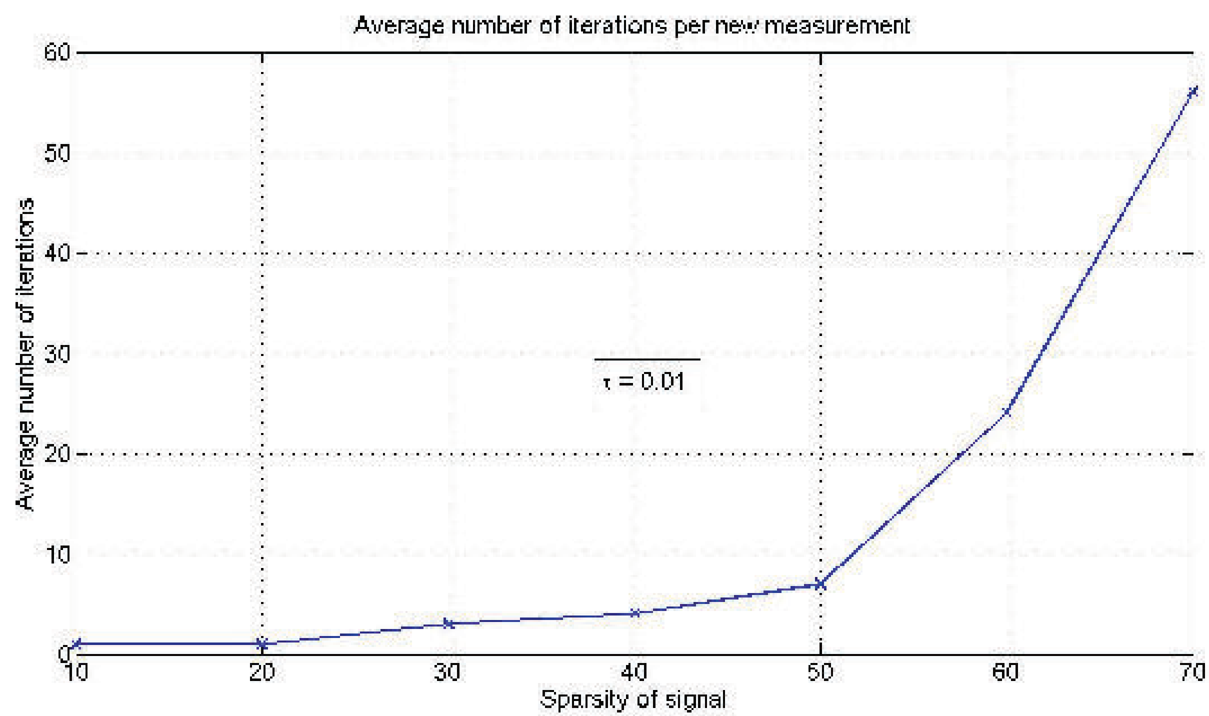
An existing element shrinks to zero

$$\theta = \min(\theta^+, \theta^-).$$

$$\boxed{\begin{aligned} \epsilon &= \epsilon_0 + \frac{\theta}{1-\theta u} \\ x^{(\epsilon)} &= x^{(\epsilon_0)} + \theta \partial x \\ \Gamma &= \{\Gamma \cup \gamma^+\} \backslash \gamma^-. \end{aligned}}$$

Performance results - CS



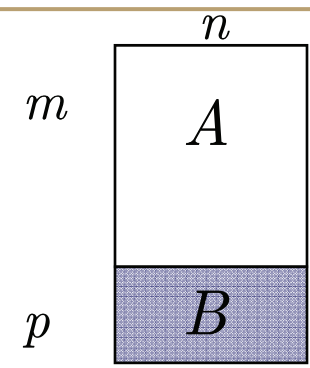


m=150

n=256

Measurement update - L^1 problems

- System model: $$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} x + \begin{bmatrix} e \\ d \end{bmatrix}$$



Updated system

- L1 Filtering

minimize $\|A\tilde{x} - y\|_1 + \|B\tilde{x} - w\|_1$

minimize $\|A\tilde{x} - y\|_1 + \epsilon \|B\tilde{x} - w\|_1$ $\quad (L^1_\epsilon)$

Homotopy parameter: change it from 0 to 1.

$\epsilon = 0$

minimize $\|\tilde{e}\|_1$

subject to $Q\tilde{e} = -Qy$.

where Q is an annihilator matrix, i.e., $QA = 0$.

- **Assumption:** *Any n x n sub-matrix of the coding matrix is non singular.*
- **Proposition:** *Using suitable coding matrix, the error estimate will have exactly n zero entries whenever original data x is not recovered.*

Optimality conditions – L^1 decoding

Add new measurement(s)

$\nu = \text{sign}(d)$

- Strong duality between primal and dual objectives

Primal

$$\begin{array}{ll} \text{minimize} & \|\tilde{e}\|_1 + \epsilon\|\tilde{d}\|_1 \\ \text{subject to} & A\tilde{x} - y = \tilde{e} \\ & B\tilde{x} - w = \tilde{d}. \end{array}$$

strong $\Longleftrightarrow$ *duality*

Dual

$$\begin{array}{ll} \text{maximize} & -\lambda^T y - \epsilon\nu^T w \\ \text{subject to} & A^T\lambda + \epsilon B^T\nu = 0 \\ & \|\lambda\|_\infty \le 1 \\ & \|\nu\|_\infty \le 1, \end{array}$$

- $x^{(\epsilon_0)}$ is a solution to L^1_ϵ at a given ϵ_0
- $e_0 := Ax^{(\epsilon_0)} - y$ is the error vector supported on index set Γ_e and $d_0 := Bx^{(\epsilon_0)} - w$ on set Γ_d.
- Using strong duality between the primal and dual objectives and feasibility conditions we get

$$\begin{array}{llll} \lambda = \text{sign}(e_0) & \text{on } \Gamma_e, & \|\lambda\|_\infty < 1 & \text{on } \Gamma_e^c \\ \nu = \text{sign}(d_0) & \text{on } \Gamma_d, & \|\nu\|_\infty < 1 & \text{on } \Gamma_d^c. \end{array}$$

Dual Update - L^1 decoding

- We have primal and dual solution at ϵ_0 denoted as $x^{(\epsilon_0)}$ and $\lambda^{(\epsilon_0)}$, and the support of $e^{(\epsilon_0)} := (Ax^{(\epsilon_0)} - y)$ is denoted as Γ.

Simple case for one new measurement.

General case is exactly same.

$$A^T\lambda^{(\epsilon_0)} + \epsilon_0 B^T\nu = 0$$

$$A^T_{\Gamma^c}\hat{\partial\lambda} + (\epsilon - \epsilon_0)B^T_{\Gamma^c}\nu = 0$$

$$\partial\lambda = \begin{cases} -(A^T_{\Gamma^c})^{-1}B^T_{\Gamma^c}z_d & \text{on } \Gamma^c \\ 0 & \text{otherwise} \end{cases}$$

- Find smallest step size θ such that one new element in λ becomes active.

$$|\lambda^{(\epsilon_0)}(j) + \theta\partial\lambda(j)| \le 1 \quad \text{for all } j \in \Gamma^c$$

$$\theta^+ = \min_{j\in\Gamma^c}\left\{\frac{1-\lambda^{(\epsilon_0)}(j)}{\partial\lambda(j)}, \frac{1+\lambda^{(\epsilon_0)}(j)}{-\partial\lambda(j)}\right\}_+$$

$$\gamma^+ = \arg\min_{j\in\Gamma^c}\left\{\frac{1-\lambda^{(\epsilon_0)}(j)}{\partial\lambda(\gamma)}, \frac{1+\lambda^{(\epsilon_0)}(j)}{-\partial\lambda(j)}\right\}_+$$

$$\epsilon = \epsilon_0 + \theta^+$$

$$\lambda^{(\epsilon)} = \lambda^{(\epsilon_0)} + \theta^+\partial\lambda.$$

Priaml Update - L^1 decoding

- Use the information from dual update to remove one existing element from Γ.

$$\begin{bmatrix} A \\ B \end{bmatrix} x^{(\epsilon_0)} - \begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} e_0 \\ d_0 \end{bmatrix} =: c$$

Primal feasibility

$$\begin{bmatrix} A \\ B \end{bmatrix}_{[\Gamma^c,:]} \partial x = \begin{bmatrix} z_{\gamma^+} \\ 0 \end{bmatrix} \begin{array}{ll} \leftarrow \text{on} & \gamma^+ \\ \leftarrow \text{on} & \Gamma^c \setminus \{\gamma^+\} \end{array}$$

Dual update

Rows indexed by Γ^c

$$A_{[\Gamma^c,:]} \partial x = \begin{cases} z_{\gamma^+} & \text{on } \gamma^+ \\ 0 & \text{on } \Gamma^c \setminus \{\gamma^+\} \end{cases}$$

Primal update

$$\partial c = \begin{bmatrix} A \\ B \end{bmatrix} \partial x$$

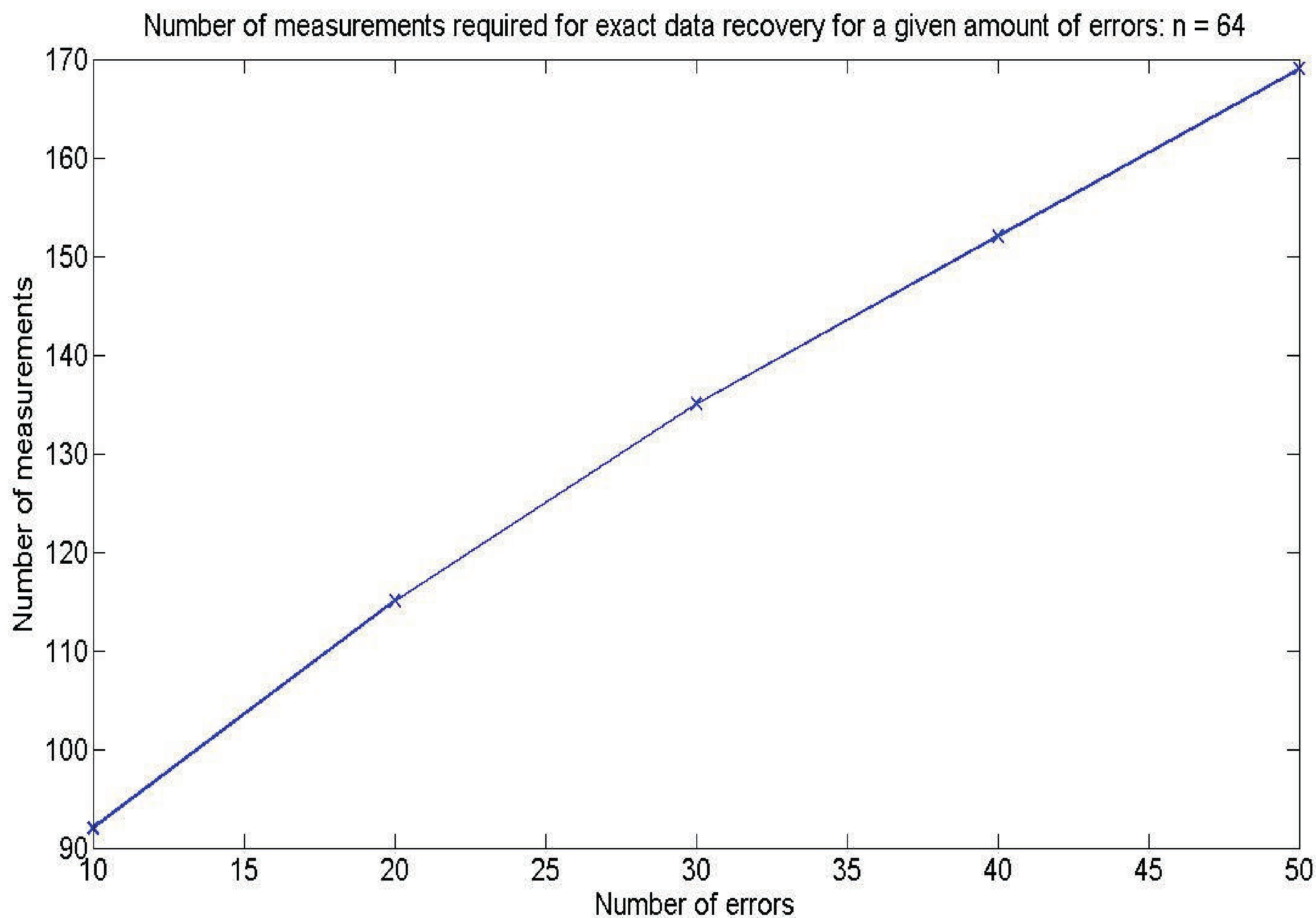
$$\left.\begin{aligned} \theta^- &= \min_{j \in \Gamma} \left(\frac{-c(j)}{\partial c(j)} \right)_+ \\ \gamma^- &= \arg\min_{j \in \Gamma} \left(\frac{-c(j)}{\partial c(j)} \right)_+ \end{aligned}\right\}$$

Smallest step such that an existing element in c shrinks to zero

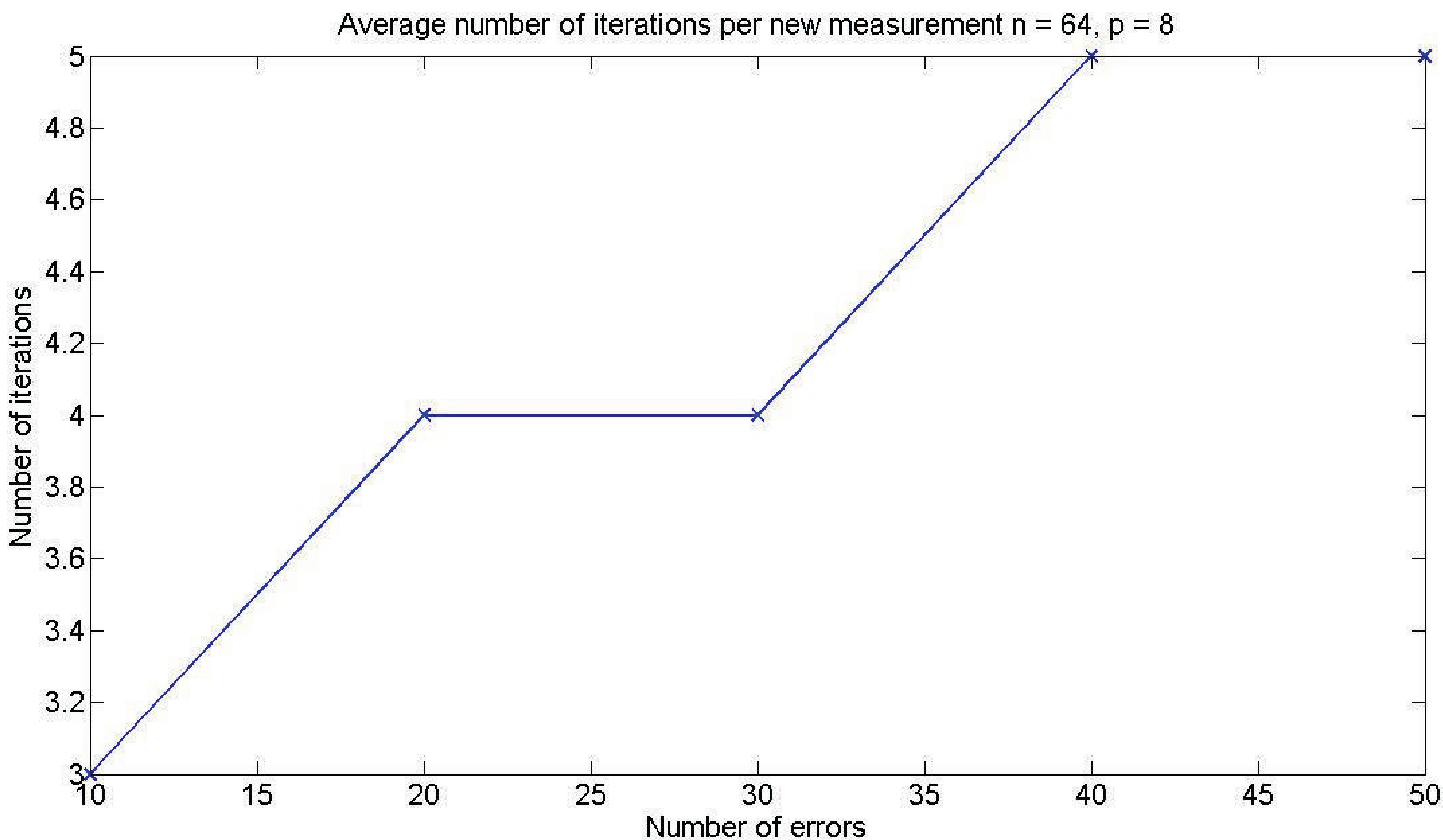
$$\boxed{\begin{aligned} x^{(\epsilon)} &= x^{(\epsilon_0)} + \theta^- \partial x \\ \Gamma &= \{\Gamma \cup \gamma^+\} \setminus \gamma^-. \end{aligned}}$$

If d becomes zero;
Lucky breakdown!

Performance results - L^1 decoding



Performance results - L^1 decoding



L^1 Decoding in the presence of noise

- Assume that the received codeword has as a small number of *gross (sparse)* errors and some small amount of *noise* in all the measurements (e.g., quantization) [Candes, Randall].
- Only gross errors – L^1 decoding
- Only noise – Least squares
- Both noise and gross errors – combined L1 and L2.
  - $y = Ax - e - z$: received codeword
  - e : sparse gross error
  - z : small noise in all the measurements.

$$\hat{x} = (A^T A)^{-1} A^T (y + \hat{e})$$

Primal

$$\begin{aligned} &\text{minimize} && \tau\|\tilde{e}\|_1 + \frac{1}{2}\|\tilde{z}\|_2^2 \\ &\text{subject to} && A\tilde{x} - y - \tilde{e} = \tilde{z} \end{aligned}$$

$$z = \lambda$$

Dual

$$\begin{aligned} &\text{maximize} && -\lambda^T y - \frac{1}{2}\|\lambda\|_2^2 \\ &\text{subject to} && A^T\lambda = 0 \\ & && \|\lambda\|_\infty \le \tau \end{aligned}$$

$$\text{minimize} \quad \tau\|\tilde{e}\|_1 + \frac{1}{2}\|Q(\tilde{e} + y)\|_2^2 \qquad Q = I - A(A^T A)^{-1} A^T$$

Measurement Update - L^1 Decoding with noise

- New system model: $$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} A \\ b \end{bmatrix} x + \begin{bmatrix} e \\ d \end{bmatrix} + \begin{bmatrix} z \\ q \end{bmatrix}.$$

new row in system

sparse error

small noise

Homotopy parameter: change it from 0 to 1.

$$\begin{aligned} \text{minimize} \quad & \tau(\|\tilde{e}\|_1 + \epsilon\|\tilde{d}\|_1) + \frac{1}{2}(\|\tilde{z}\|_2^2 + \|\tilde{q}\|_2^2) \\ \text{subject to} \quad & A\tilde{x} - \tilde{e} - y = \tilde{z} \\ & b\tilde{x} - \tilde{d} - w = \tilde{q} \end{aligned}$$

$$\begin{aligned} \text{maximize} \quad & -\lambda^T y - \nu^T w - \frac{1}{2}(\|\lambda\|_2^2 + \|\nu\|_2^2) \\ \text{subject to} \quad & A^T\lambda + b^T\nu = 0 \\ & \|\lambda\|_\infty \le \tau \\ & \|\nu\|_\infty \le \epsilon\tau \end{aligned}$$

$$G := \begin{bmatrix} A \\ b \end{bmatrix}, P = I - G(G^TG)^{-1}G^T$$

$$\boxed{\text{minimize} \quad \tau(\|\tilde{e}\|_1 + \epsilon\|\tilde{d}\|_1) + \frac{1}{2}\left\| P\left(\begin{bmatrix} \tilde{e} \\ \tilde{d} \end{bmatrix} + \begin{bmatrix} y \\ w \end{bmatrix}\right)\right\|_2^2}$$

Homotopy scheme - L^1 Decoding with noise

- Optimality conditions similar to Lasso

$$\text{minimize} \quad \tau(\|\tilde{e}\|_1 + \epsilon\|\tilde{d}\|_1) + \frac{1}{2}\left\| P\left(\begin{bmatrix} \tilde{e} \\ \tilde{d} \end{bmatrix} + \begin{bmatrix} y \\ w \end{bmatrix} \right) \right\|_2^2$$

- At any given ϵ and τ the optimality conditions are given as:

$$\left\| P\Big(\underbrace{\begin{bmatrix} \tilde{e} \\ \tilde{d} \end{bmatrix}}_{c} + \underbrace{\begin{bmatrix} y \\ w \end{bmatrix}}_{s} \Big) \right\|_\infty \preceq \begin{bmatrix} \tau \\ \epsilon\tau \end{bmatrix}$$

$$\boxed{\partial c = \begin{cases} -P^{-1}_{[\Gamma,\Gamma]} \begin{bmatrix} 0 \\ z_d \end{bmatrix} & \text{on } \Gamma \\ 0 & \text{elsewhere} \end{cases} \qquad \Gamma_e \quad \Gamma_d}$$

(L1). $P_{[\Gamma_e,:]}(c+s) = -\tau z_e$
(L2). $P_{[\Gamma_d,:]}(c+s) = -\epsilon\tau z_d$
(L3). $\|P_{[\Gamma^c,:]}(c+s)\|_\infty < \tau$

where $\Gamma = \{\Gamma_e \cup \Gamma_d\}$ is the support of $c := \begin{bmatrix} e \\ d \end{bmatrix}$, z_e and z_d are the corresponding sign sequences.

Homotopy scheme - L^1 Decoding with noise

- Choose appropriate step size such that either one element becomes active in c or an element leaves its support.

- If d shrinks to zero: Lucky breakdown.

$$\boxed{\begin{aligned}\epsilon &= \epsilon_0 + \frac{\theta}{\tau}\\ c^{(\epsilon)} &= c^{(\epsilon_0)} + \theta \partial c.\end{aligned}}$$

$$\| \underbrace{P(c+s)}_{p_k} + \theta \underbrace{P\partial c}_{d_k} \|_\infty \preceq \begin{bmatrix} \tau \\ \epsilon\tau \end{bmatrix}$$

$$|p_k(j) + \theta d_k(j))| = \tau \qquad \text{for all } j \in \Gamma_e$$

$$|p_k(j) + \theta d_k(j))| = \tau\epsilon \qquad \text{for all } j \in \Gamma_d$$

$$|p_k(j) + \theta d_k(j))| \leq \tau \qquad \text{for all } j \in \Gamma^c$$

$$\theta^+ = \min_{j \in \Gamma^c} \left(\frac{\tau - p_k(j)}{d_k(j)}, \frac{\tau + p_k(j)}{-d_k(j)} \right)_+$$

$$j^+ = \arg\min_{j \in \Gamma^c} \left(\frac{\tau - p_k(j)}{d_k(j)}, \frac{\tau + p_k(j)}{-d_k(j)} \right)_+$$

$$\theta^- = \min_{j \in \Gamma} \left(\frac{-c^{(\epsilon_0)}(j)}{\partial c(j)} \right)_+$$

$$j^- = \arg\min_{j \in \Gamma} \left(\frac{-c^{(\epsilon_0)}(j)}{\partial c(j)} \right)_+$$

$$\theta = \min(\theta^+, \theta^-).$$

Summary

- Homotopy method for Compressive sensing
  - Very fast for small scale problems (where explicit matrices can be used) - rank one update at each step.
  - Ability for dynamic update of solution with new measurements (both Lasso and Dantzig Selector).
- Homotopy for L1 decoding
  - L1 decoding with sparse errors
  - L1 decoding with sparse errors and small noise
  - Fast update with sequential observations.
- Least squares → Recursive least squares (RLS)
  - rank update
- L1 problems → Dynamic update
  - rank update along with homotopy

Thankyou !

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Questions