Optical Bistable Switching With Kerr Nonlinear Materials Exhibiting a Finite Response Time in Two-Dimensional Photonic Crystals

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ABSTRACT

Effect of relaxation time on the performance of photonic crystal optical bistable switches based on Kerr nonlinearity is discussed. This paper deals with optical pulses with the duration of about 50 ps. In such cases the steady state response of the optical device can be used to approximate the pulse evolution if the nonlinearity is assumed instantaneous, hence analytical solutions such as the coupled mode theory can be used to obtain the time evolution of the electromagnetic fields. However if the relaxation time of the material nonlinear response is also considered, changes in the power levels and in the shape of the hysteresis loop is observed. In this case, we use the nonlinear finite difference time domain method (NL-FDTD) to follow the system dynamics and get the bistability hysteresis loop. Codes are developed to analyze the instantaneous Kerr materials and the Kerr materials with finite response times. Depending on the material, the relaxation times of the order of 1-10fs should be considered in studying bistability to obtain the right shape of the output pulses. It is observed that the relaxation leads to larger input power and threshold and hence degrades the performance of the switch in pulse shaping.

Keywords: nonlinear optics, photonics crystals, finite difference time domain method, optical bistability, optical switching

1. INTRODUCTION

Optical bistability is believed to be a beneficial candidate to speed up optical signal processing and communications. So much effort has been devoted during last years to investigate optical bistable elements1 and different structures have been proposed to realize them, e.g. microring and photonic crystals.

Nonlinearity is an inevitable requirement of optical bistability. Unfortunately nonlinear effects are very weak in photonics compared to electronics so one should consume more power. Another consideration is the nonlinear process speed which omits effects such as two photon absorption from over-gigahertz switching applications2. Kerr effect, however, seems to be the proper candidate at such speeds. Optical Kerr effect can be considered to have a relaxation time in the order of 1-10fs depending on the material3. However most investigations ignore this fact and assumed the Kerr effect to be instantaneous4. This approximation, however, can change the shape of the power characteristic, have an influence on the power thresholds and affect the system dynamics which is discussed in this paper.

It is useful to remind that the linear material dispersion is not included in this paper. Clearly, materials might be found that their dispersive properties are stronger than their nonlinear properties. Also, only one type of nonlinear dispersion is taken into account. One could use other kinds of dispersion to investigate the bistable behavior of the structure. However, the simplicity of this phenomenological model and its applicability to some materials urged us to use it.

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Modeling nonlinearity has been a matter of interest for many years. Different numerical methods have been extended to be able to model nonlinearity among which, FDTD\textsuperscript{7,8}, the Finite Element Method (FEM)\textsuperscript{9,10} and the Beam Propagation Method (BPM)\textsuperscript{11} are the most popular. BPM is normally used to model light-matter interaction in the bulk material or in the waveguide structures. FEM can be used to simulate more sophisticated structures e.g. the ones with resonators; however, it cannot generally be useful to track the time evolution of the waves when the system is nonlinear because in that case, the coupling between different frequencies necessitates simultaneous consideration of various frequencies. Furthermore, FEM usually needs a huge amount of memory since it deals with very large matrices.

One of the most powerful tools to investigate bistability is the coupled mode theory (CMT). The method provides us with the analytical solution of the nonlinear structure. However, the results of CMT exclude the transient effects and cannot be used in the case of extremely short pulses. On the other hand, the other popular method, the NL-FDTD method is based on the time stepping of the electromagnetic fields, so it can be used to investigate the dynamics of a bistable device. The main disadvantage of the latter method is the computational burden and therefore the long time it takes.

The contents of this paper are arranged in the following order: First we introduce the formulation used to model instantaneous Kerr effect and the Kerr medium with the finite response time. Then the structure is introduced and finally the results of FDTD simulation are discussed.

### 2. FORMULATION

FDTD is one of the most popular full-numerical methods to simulate nonlinearity and dispersion structure. It can be used to model quite sophisticated geometries in different dimensions. Since it is a time domain approach, one can obtain time evolution of the fields and quantities such as power flow in the desired places in the structure. This property is specifically useful in studying transients; however, acceptable approximations of the steady-state behavior of the device can be achieved as well by letting the FDTD simulation run for a long time. Therefore, it has been proved to be quite beneficial as a benchmark to study optical bistability\textsuperscript{4-6,12,13}.

FDTD modeling of nonlinear and dispersive media has attracted much interest in the last decades. Some methods have been proposed to model such to achieve this goal among which the Piecewise Linear Recursive Convolution (PLRC)\textsuperscript{14,15} method, Auxiliary Differential Equation (ADE)\textsuperscript{16,17} method and Z-transform\textsuperscript{18,19} method are the most popular. In this manuscript, we use ADE formulation because of its stability and ease of implementation.

Faraday’s and Ampère’s laws in a nonmagnetic source-free medium are in the following form:

\[
\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t},
\]

\[
\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t}
\]

(1)

(2)

For an instantaneous Kerr medium the relation between displacement and electric field takes the form:

\[
\mathbf{D}(\mathbf{r}, t) = \varepsilon_0 [\varepsilon_r + \chi_0^{(3)} | \mathbf{E}(\mathbf{r}, t) |^2] \mathbf{E}(\mathbf{r}, t)
\]

(3)

In a time stepping algorithm, we update \( \mathbf{H}(\mathbf{r}, t) \) from (1) and \( \mathbf{D}(\mathbf{r}, t) \) from (2). \( \mathbf{E}(\mathbf{r}, t) \) is obtained by solving (3). This can be done either iteratively or analytically in FDTD\textsuperscript{20}. The analytical solution, however, is a bit faster than the numerical solution. We used the analytical method to simulate the instantaneous Kerr medium.

To account for the finite response time, we use the following equations according to\textsuperscript{3}:

\[
\mathbf{D}(\mathbf{r}, t) = \varepsilon_0 [\varepsilon_r + \chi] \mathbf{E}(\mathbf{r}, t),
\]

(4)

\[
\tau \frac{\partial \chi}{\partial t} + \chi = \chi_0^{(3)} | \mathbf{E} |^2,
\]

(5)

where \( \tau \) is the relaxation time and is assumed 1-10 fs. If the pulse is very wide in the time domain, i.e. much longer than the relaxation time, the Kerr medium may be assumed to have an instantaneous response. In this way the bistability
hysteresis loops in [4] seem to be almost correct since they exhibit the steady state response of the device. However the results of our simulations show that the assumption of instantaneous nonlinear response causes dramatic changes in the device characteristics. Note that in the general case, function $\chi$ in (4) and (5) can be expressed as

$$\chi(t) = \chi^{(3)}(t) * E^2(t).$$  \hspace{1cm} (6)

where * stands for convolution. One can update $\chi$ to obtain the following update relation

$$\chi^{n+1} = A(E^n)^n + B\chi^n + C\chi^{n-1}. \hspace{1cm} (7)$$

Different types of nonlinear dispersion can be modeled through obtaining the coefficients $A$, $B$ and $C$ in the latter relation.

**3. STRUCTURE AND SIMULATION PARAMETERS**

Two main geometries have been proposed to realize optical bistable elements in two dimensions: the direct coupled and the side coupled (fig. 1). Both structures include a waveguide to provide a medium for transmitting power and a resonator for making resonance. In the direct-coupled structure, a fraction of input power is transmitted to the output through resonant tunneling, however, in the side-coupled configuration, resonance blocks the input power in the vicinity of resonant frequency. The basics of operation of both structures are the same, so solving the problem for one of them will guide us to the response of the other one.

![Figure 1. (Left): direct-coupled structure and (right): side-coupled structure.](image)

We chose the direct coupled structure of fig.2 and the results of this paper are obtained for this geometry. The structure and the parameters are the same as in [4]. The bulk is formed by rods of $r=0.25a$ where $a$ is the lattice constant. The radius of the rod in the middle of the structure is increased to $5/3r$ to form a resonator and the small rods in the waveguides are of radius $r/3$ to slow the input wave. All rods have permittivity of $\varepsilon_r=12.25$ and the background permittivity is $\varepsilon_b=2.25$. The grey rods are considered nonlinear and the TM (E-polarized) mode is considered. The PML layer consists of 20 layers with polynomial graded conductivity profile. The nonlinear coefficient is $1\times10^{-17}$ m$^2$/W and the relaxation time of the nonlinear material is taken to be only 1/f. A Gaussian pulse with temporal envelop distribution of $\exp[-(t-t_0)^2/(2\tau^2)]$ is considered as the input where $t_0$ is the simulation mid-time. The carrier wavelength is almost 1.55$\mu$m, $\Delta x = \Delta y = \Delta z = \Delta t/c \times \sqrt{2}$ and the simulation is run for $4\times10^5$ time steps. The simulation is time consuming since the input power should vary slowly to form the hysteresis curve. In fact each point in the bistability curve should refer to a continuous wave input. However if the input field envelope changes very slowly, it can be a good approximation of the continuous wave. The structure shows optical bistability and the field profiles corresponding to the two output levels are shown in fig. 2. The simulations are performed at $\delta=8.4$ where $\delta$ is the relative frequency detuning from the cavity resonant frequency

$$\delta = \frac{\omega - \omega_0}{\gamma}; \hspace{1cm} (8)$$

in which $\omega$ is the carrier frequency and $\omega_0$ is the resonant frequency. The parameter $\gamma$ shows width of resonance

$$\gamma = \frac{\omega_0}{2Q}. \hspace{1cm} (9)$$

Note that if $\delta$ is taken very large, the power threshold of the device will be very high. On the other hand if it is very small, i.e. operation is performed very close to resonance, bistability will disappear.
4. RESULTS AND DISCUSSIONS

Fig. 3 shows the normalized values for the envelope of the input and output powers launched into the structure considering the finite response time. The bistable behavior is almost obtained but the input power is much higher than the case of instantaneous Kerr material. The reason is that the delay between occurrence of the field maxima and the nonlinear response avoids the term $\chi E(r,t)$ in (4) from growing very large. The obtained hysteresis loop is depicted in Fig. 4. The power is plotted for thickness of 1μm. $P_0$ is the characteristic power of the device as defined in [4]. The power is much higher than the consumed power for the instantaneous Kerr model which is in contradiction with the desire of having a low power switch. In fact, at the power threshold of the instantaneous Kerr model, the second model is almost linear and bistability happens for power values far more than that. Not only the power is unacceptably high, the contrast ratio – which is defined as the ratio of difference between the output power levels to the difference between the input power levels- is also very low as a result of increase in the input power. Note that the relaxation time is only 1/fs in our simulations. For larger values of $\tau$, the bistable behavior is expected to degrade even more. The rising side of the loop is not as sharp as it is in the instantaneous Kerr model and the loop tends to close. Field profile for the ON state is shown in the inset of fig.4. Almost no power is transmitted to the output compared to the case of instantaneous Kerr material.

![Figure 3](image-url)
Figure 4. The approximate hysteresis loop obtained from the NL-FDTD method. The envelop of input pulse was of the form $\exp\left(-\frac{(t-t_0)}{t_0}^2 \times 6\right)$.

5. CONCLUSIONS

A direct-coupled photonic crystal switch was analyzed with the aid of a here-developed NL-FDTD code in the two cases of instantaneous nonlinear Kerr material and the Kerr material with a relaxation time of the $1/\text{fs}$. The relaxation was found to cause the power levels of the device and the threshold power to increase. The transmission is also reduced to very low values, hence considering the finite response time shows that low power switching of ultra-short pulses with the width of about $50 \text{ ps}$ is an extremely hard job and needs more sophisticated considerations.

REFERENCES