Spring 2009
ECE 4601: Assignment 1

- Date Assigned: January 13, 2009.
- Date Due: January 22, 2009.

1. Let a pdf be given by $f(x) = Ke^{-b|x|}, -\infty \le x \le \infty$, where K and b are positive constants.
   - Find the value required for K in terms of b.
   - Find the variance σ_X^2 in terms of b.
   - Derive the cumulative distribution function $F_X(x)$?

2. Let Θ be uniformly distributed on $[0, \pi]$ and let random variables X and Y be defined by $X = \cos\Theta$ and $Y = \sin\Theta$. Show that X and Y are uncorrelated but they are not independent.

3. let $y = ax_1 + bx_2$, where x_1 and x_2 are independent random variables. Show that
   - Show that $\mu_y = a\mu_{x_1} + b\mu_{x_2}$.
   - Show that $\sigma_y^2 = a^2\sigma_{x_1}^2 + b^2\sigma_{x_2}^2$.

   Hint: Use the ensemble average operator notation, i.e., $\mu_y = E[y], \sigma_y^2 = E[(y - \mu_y)^2]$. No integrals are required.

4. Consider a zero-mean Gaussian random variable with variance σ^2. This random variable is pass through a system whose input-output relation is given by $y = g(x)$. Find the pdf of the output random variable in each of the following cases in terms of A, x_0, μ, and σ. Use delta functions if needed in your pdf.
   - square-law device, $g(x) = x^2$.
   - limiter
   $$g(x) = \begin{cases} -b, & x \le -b \\ b, & x \ge b \\ x, & |x| \le b \end{cases}$$
   - hard limiter
   $$g(x) = \begin{cases} a, & x > 0 \\ 0, & x = 0 \\ -a, & x < 0 \end{cases}$$

5. *Challenging* Let the random vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ be jointly Gaussian with mean $\boldsymbol{\mu}_x = (\mu_1, \mu_2, \ldots, \mu_n)^T$ and covariance matrix $\boldsymbol{\Lambda}$. Define a new random vector $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)^T$, such that

$$\mathbf{Y} = \mathbf{A}\mathbf{X}$$

where $\mathbf{A}$ is a constant matrix. Using the fact that linear functions of jointly Gaussian random variables are themselves jointly Gaussian, find the mean and covariance matrix of $\mathbf{Y}$.