Spring 2010
ECE 4601: Assignment 1

- Date Assigned: January 14, 2010.
- Date Due: January 26, 2010.

1. Suppose $r(t) = \alpha \cos(2\pi f_o t) + \beta \cos(2\pi(f_o + f_d)t)$. Show that

$$r(t) = A(t) \cos(2\pi f_o t + \phi(t))$$

where

$$\begin{aligned} A(t) &= \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \cos(2\pi f_d t)} \\ \phi(t) &= \mathrm{Tan}^{-1} \frac{\beta \sin(2\pi f_d t)}{\alpha + \beta \cos(2\pi f_d t)} \end{aligned}$$

2. A customer desires a communication system that is capable of conveying 60 kilobits per second through a channel bandwidth of 10 kHz. The customer can achieve a received signal power of 1 picowatt. The channel noise is 10^{-19} watts per hertz of bandwidth. Should your company submit a bid to build the communication system?

3. Consider a binary discrete memoryless channel, having the inputs $X \in \{0, 1\}$ and outputs $Y \in \{0, 1\}$, such that

$$\begin{aligned} P(Y = 0|X = 0) &= 0.7 \qquad & P(Y = 0|X = 1) &= 0.1 \\ P(Y = 1|X = 0) &= 0.3 \qquad & P(Y = 1|X = 1) &= 0.9 \\ P(X = 0) &= 0.4 \qquad & P(X = 1) &= 0.6 \end{aligned}$$

   a) Find the probabilities $P(Y = 0)$ and $P(Y = 1)$ at the channel output.
   b) Find the probability that a 1 was sent given that a 1 was received.
   c) Repeat parts a) and b) if two such channels are concatenated together in a serial fashion.

4. Let Θ be uniformly distributed on $[0, \pi]$ and let random variables X and Y be defined by $X = \cos\Theta$ and $Y = \sin\Theta$. Show that X and Y are uncorrelated but they are not independent.

5. let $y = ax_1 + bx_2$, where x_1 and x_2 are independent random variables. Show that

   - Show that $\mu_y = a\mu_{x_1} + b\mu_{x_2}$.
   - Show that $\sigma_y^2 = a^2\sigma_{x_1}^2 + b^2\sigma_{x_2}^2$.

   Hint: Use the ensemble average operator notation, i.e., $\mu_y = E[y], \sigma_y^2 = E[(y - \mu_y)^2]$. *No integrals are required.*

6. Consider a zero-mean Gaussian random variable with variance σ^2. This random variable is pass through a system whose input-output relation is given by $y = g(x)$. Find the pdf of the output random variable in each of the following cases in terms of A, x_0, μ, and σ. Use delta functions if needed in your pdf.

   - square-law device, $g(x) = x^2$.
   - limiter

$$g(x) = \begin{cases} -b, & x \leq -b \\ b, & x \geq b \\ x, & |x| \leq b \end{cases}$$

   - hard limiter

$$g(x) = \begin{cases} a, & x > 0 \\ 0, & x = 0 \\ -a, & x < 0 \end{cases}$$