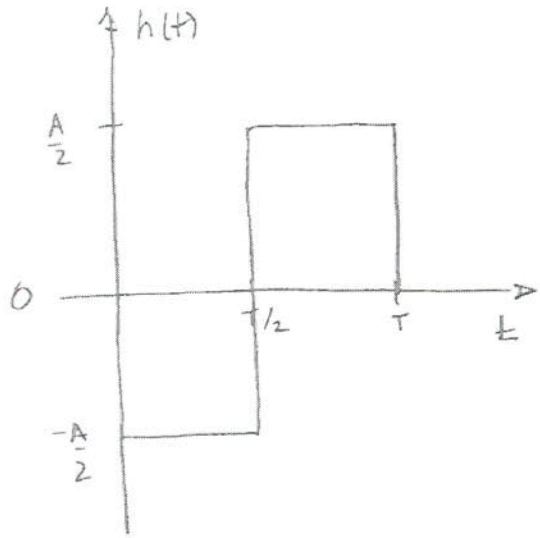
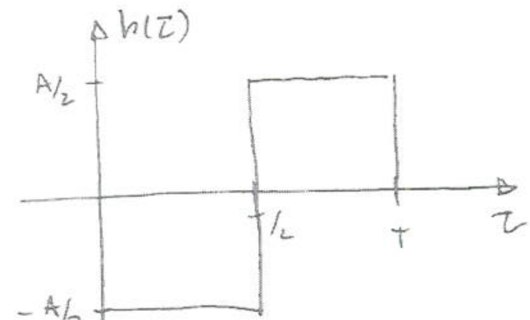
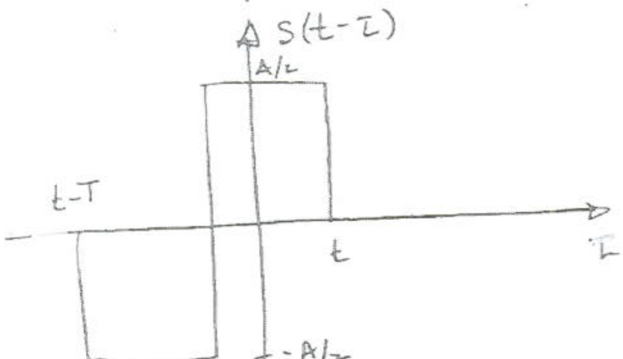
ECE 4601 Homework #3 Solutions.

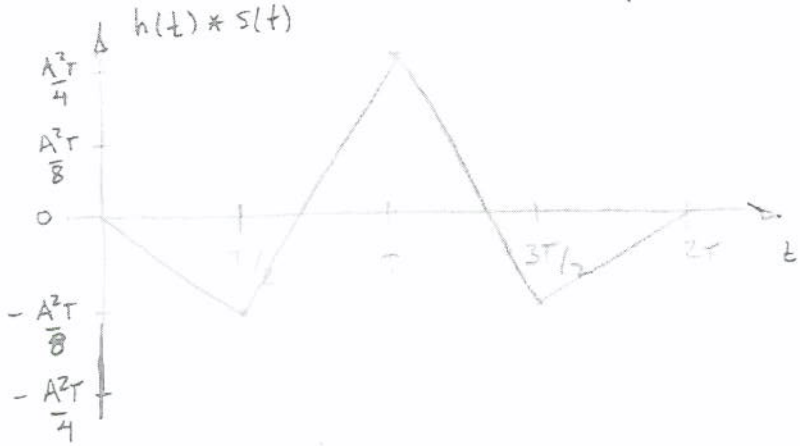
1 a) $h(t) = s(T-t)$



b) easiest to use graphical convolution

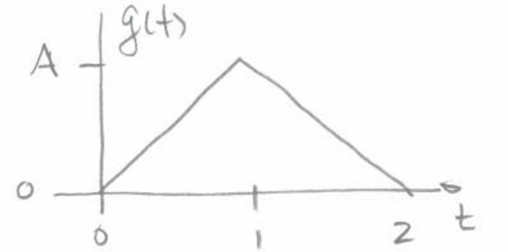






Peak output is $\frac{A^2T}{4}$, the energy in $s(t)$.

2a) $g(t) = A\,\Lambda(t-1)$

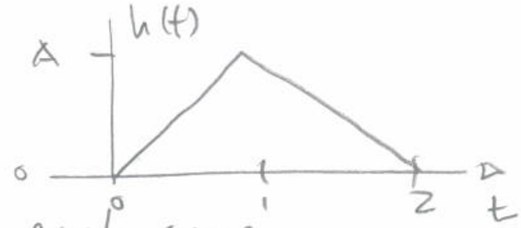


$h(t) = g(2-t)$

$= A\Lambda(2-t-1)$

$= A\Lambda(1-t)$

$= A\Lambda(t-1)$

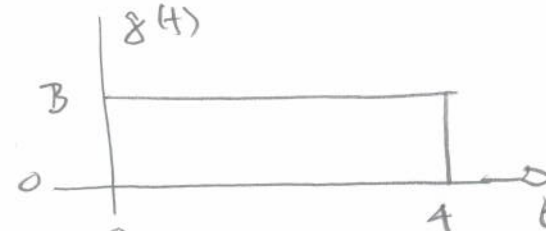


$\eta_{max} = \frac{2E}{N_0} = 2E$ in each case

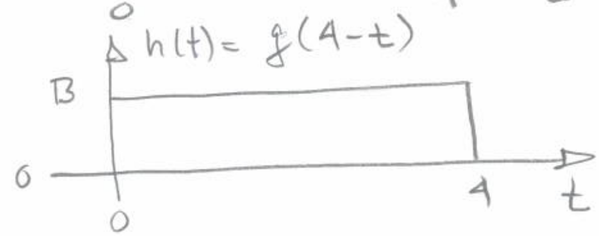
$E = 2A^2\int_0^1 t^2\,dt = 2A^3\frac{t^3}{3}\Big|_0^1 = \frac{2}{3}A^2$

$\therefore\ \eta_{max} = \frac{4A^2}{3}$

b) $g(t) = B\,\mathrm{rect}\left(\frac{t-2}{4}\right)$

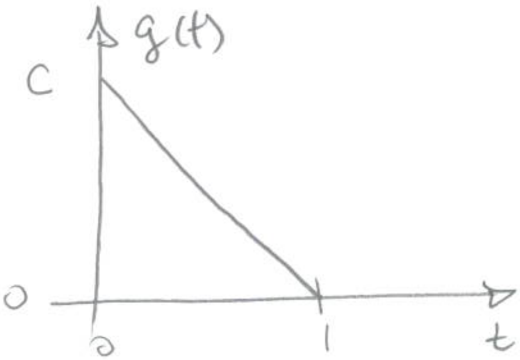


$h(t) = B\,\mathrm{rect}\left(\frac{t-2}{4}\right)$



$E = 4B^2 \Rightarrow \eta_{max} = 8B^2$

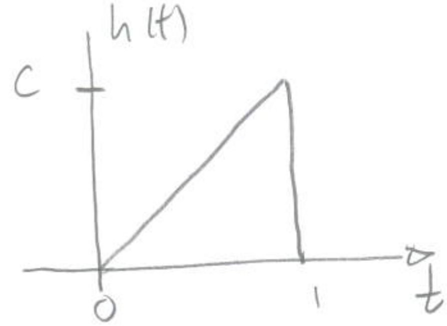
c) $g(t) = C\Lambda(t)\,u(t)$



$h(t) = g(1-t)$

$= C\Lambda(1-t)\,u(1-t)$

$= C\Lambda(t-1)\,u(1-t)$



$E = C^2 \int_0^1 (1-t)^2\,dt$

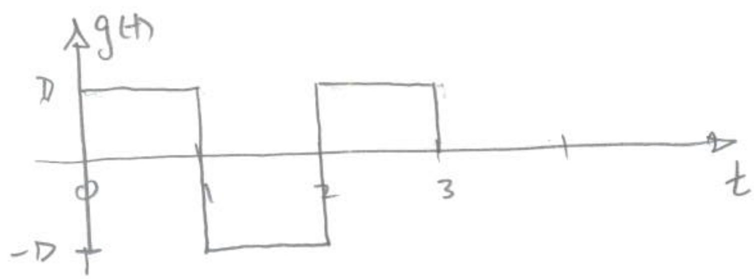
$= C^2 \int_0^1 (1 - 2t + t^2)\,dt$

$= C^2 \left(t - \frac{2t^2}{2} + \frac{t^3}{3} \right)\Big|_0^1$

$= C^2 \left(1 - 1 + \frac{1}{3} \right)$

$= \frac{C^2}{3} \implies \eta_{max} = \frac{2C^2}{3}$

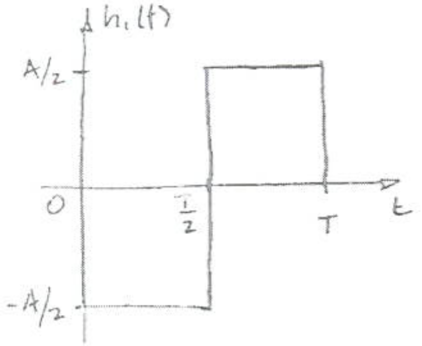
d) $g(t) = D\left[u(t) - 2u(t-1) + 2u(t-2) - u(t-3)\right]$

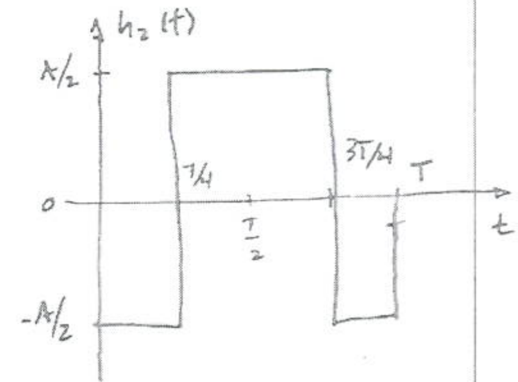


$h(t) = g(3-t) = g(t)$

$E = 3D^2 \implies \eta_{max} = 6D^2$

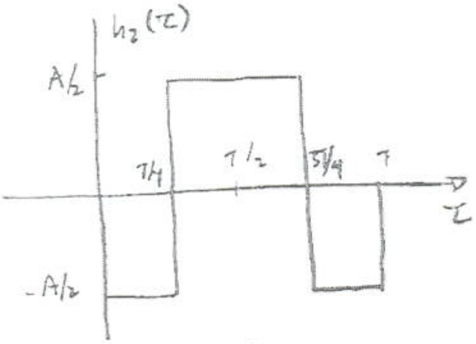
3 a) The matched filters are $h_i(t) = s_i(T-t)$, $i = 1, 2$



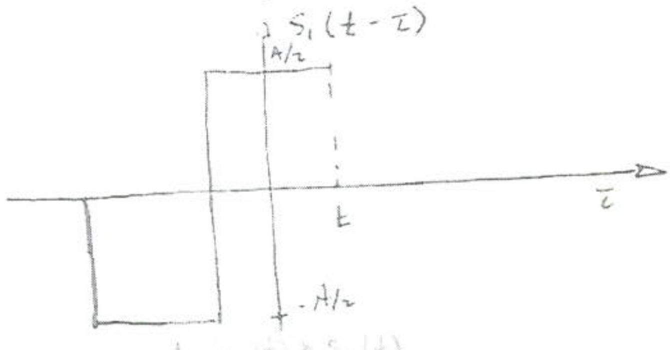


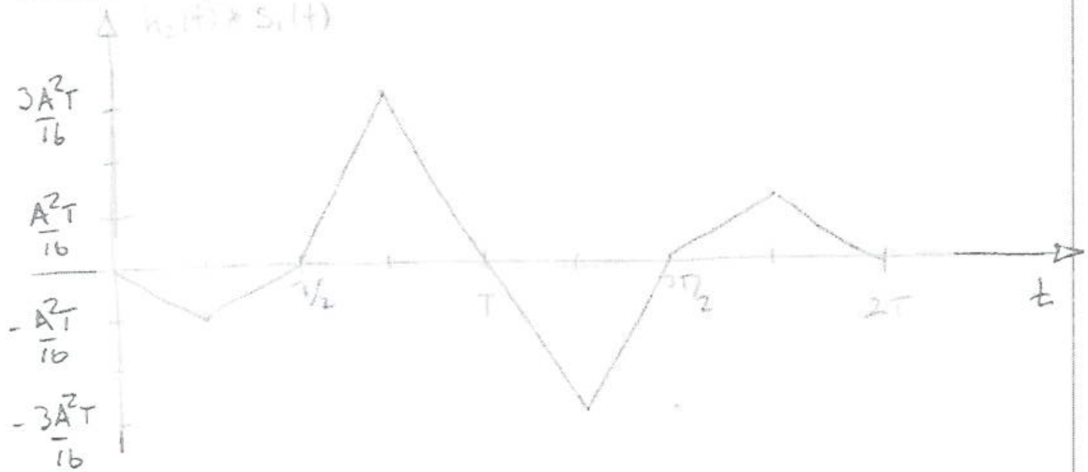
b) Consider the response of $h_2(t)$ to the input $s_1(t)$

i)

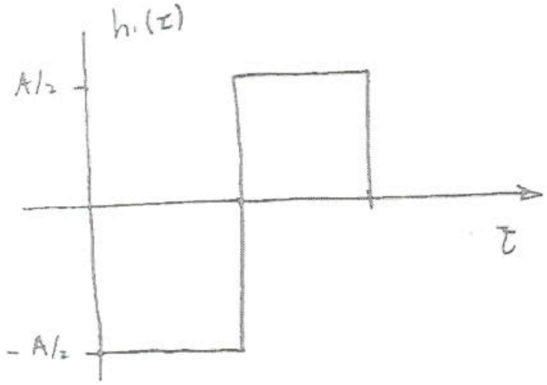


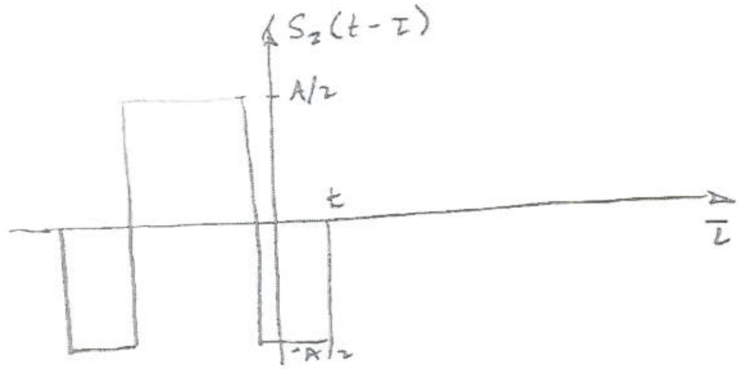
Graphical convolution is the easiest

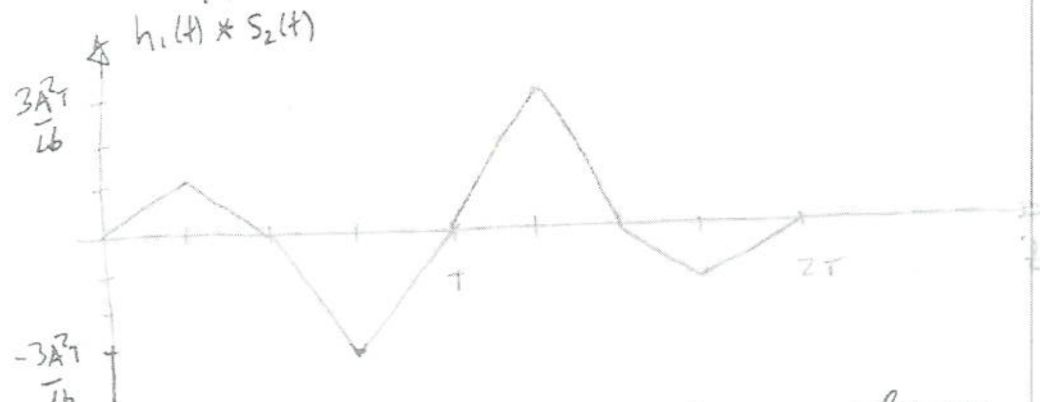




ii)







Consider a set of orthogonal waveforms $s_i(t)$, $i = 1, \dots M$. The matched filters are
$h_i(t) = s_i(T-t)$ $i = 1, \dots M$.

At $t = T$ we have

$$\int_0^T s_i(\tau)\, h_j(T-\tau)\, d\tau = \begin{cases} E_i & i = j \\ 0 & i \neq j \end{cases}$$

$E_i = \int_0^T s_i^2(t)\,dt$ is energy in $s_i(t)$.

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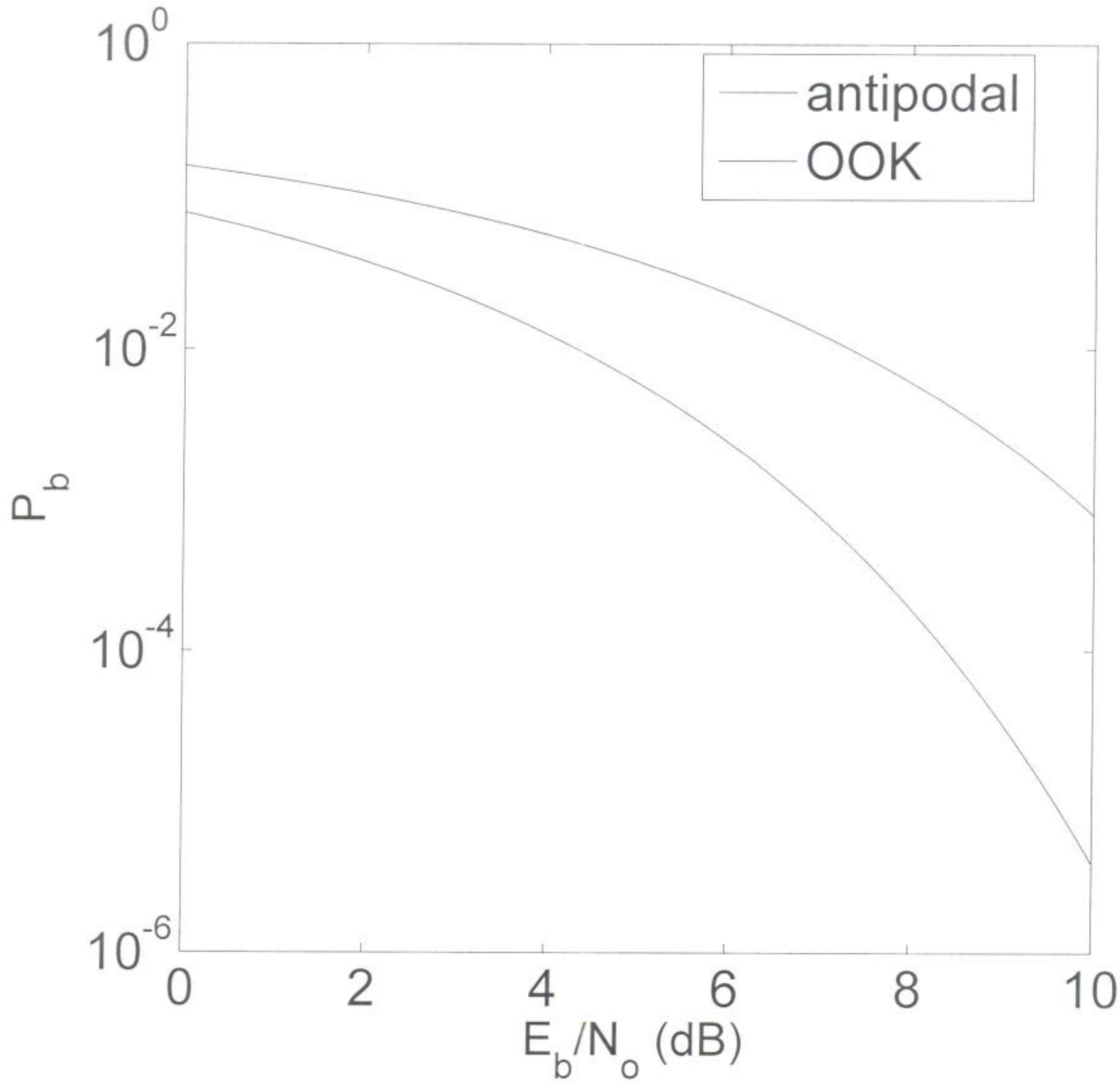
4.

```
%PLOT of error prob

%clear beta t rc rootrc
%clear all;
%close all;

%Input variables%
ebno = 1:0.01:10;
ebnodb = 10*log10(ebno);
pbbpsk = 0.5*erfc(sqrt(ebno));
pbook = 0.5*erfc(sqrt(ebno/2.0));

plot(ebnodb,pbbpsk,'k-');

hold on

plot(ebnodb,pbook,'r-');

xlabel('E_b/N_o (dB)','fontsize',18)
ylabel('P_b','fontsize',18)
```



5a) $s(t) = \begin{cases} \frac{At}{T}, & 0 \le t \le T \\ 0, & \text{else} \end{cases}$

a) Matched filter is

$$h(t) = s(T-t) = \begin{cases} \frac{A(T-t)}{T}, & 0 \le t \le T \\ 0, & \text{else} \end{cases}$$

Matched filter output is $y(t) = s(t) * h(t)$

we can write

$$s(t) = \frac{At}{T}\left[u(t) - u(t-T)\right]$$

$$h(t) = \frac{A(T-t)}{T}\left[u(t) - u(t-T)\right]$$

$$y(t) = \int_{-\infty}^{\infty} \frac{A\tau}{T}\left[u(\tau) - u(\tau - T)\right]\frac{A(T-t+\tau)}{T}\left[u(t-\tau) - u(t-\tau-T)\right]d\tau$$

For $0 \le t \le T$

$$y(t) = \int_0^t \frac{A\tau}{T} \cdot \frac{A(T-t+\tau)}{T}\, d\tau$$

$$= \frac{A^2}{T^2}\int_0^t \left[(T-t)\tau + \tau^2\right]d\tau$$

$$= \frac{A^2}{T^2}\left[(T-t)\frac{t^2}{2} + \frac{t^3}{3}\right]$$

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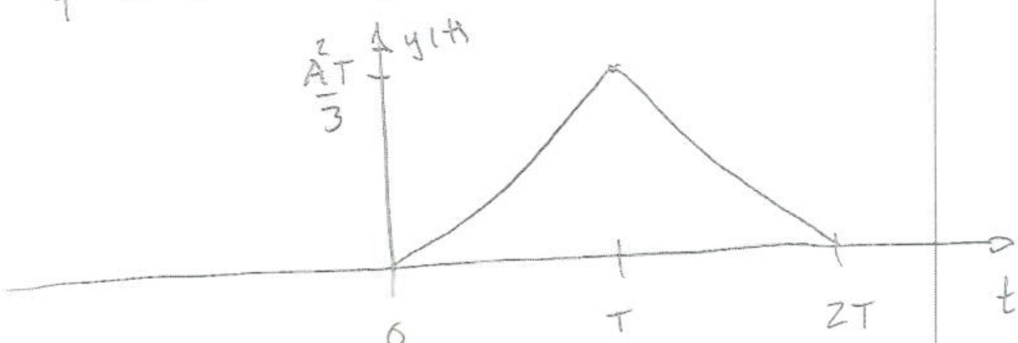
For $T \le t \le 2T$

$$y(t) = \int_{t-T}^{T} \frac{A\tau}{T} \cdot \frac{A(T-t+\tau)}{T}\, d\tau$$

$$= \frac{A^2}{T^2} \int_{t-T}^{T} \left[(T-t)\tau + \tau^2 \right] d\tau$$

$$= \frac{A^2}{T^2} \left[(T-t)\frac{\tau^2}{2} + \frac{\tau^3}{3} \right]_{t-T}^{T}$$

$$= \frac{A^2}{T} \left[\frac{T^3}{3} + \frac{(T-t)T^2}{2} + \frac{(t-T)^3}{6} \right]$$

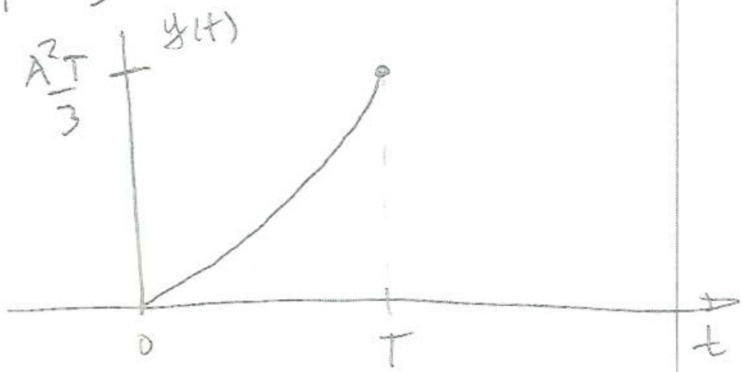


b) For correlator

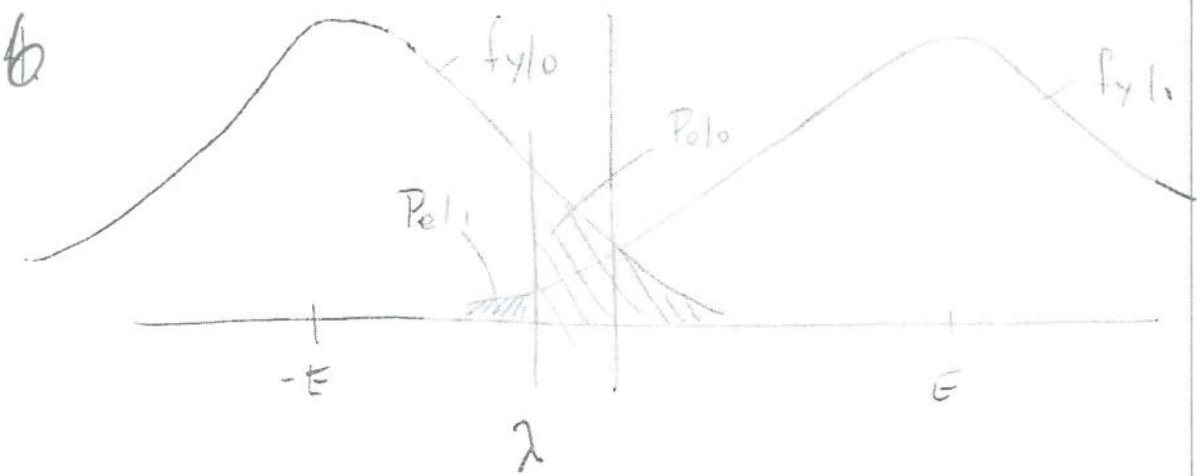
$$y(t) = \int_0^t s^2(t)\,dt$$

$$= \int_0^t \frac{A^2 t^2}{T^2}\, dt$$

$$= \frac{A^2}{T^2}\frac{t^3}{3}$$



b



Let $p = P(\text{"1"})$
$1-p = P(\text{"0"})$

Then

$$P_e = P_{e|1}\, P(\text{"1"}) + P_{e|0}\, P(\text{"0"})$$

$$= Q\left(\frac{E+\lambda}{\sigma_W}\right)p + Q\left(\frac{E-\lambda}{\sigma_W}\right)(1-p)$$

$$= p\int_{\frac{E+\lambda}{\sigma_W}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx + (1-p)\int_{\frac{E-\lambda}{\sigma_W}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$$

Solve $\frac{dP_e}{d\lambda} = 0$

Use identity $$\frac{d}{da}\int_p^q f(x,a)\,dx = \int_p^q \frac{\partial}{\partial a}[f(x,a)]\,dx + f(q,a)\frac{dq}{da} - f(p,a)\frac{dp}{da}$$

So

$$\frac{dP_e}{d\lambda} = -p \cdot \frac{1}{\sqrt{2\pi}\,\sigma_w} e^{-\frac{1}{2}\left(\frac{E+\lambda}{\sigma_w}\right)^2} + (1-p)\frac{1}{\sqrt{2\pi}\,\sigma_w} e^{-\frac{1}{2}\left(\frac{E-\lambda}{\sigma_w}\right)^2} = 0$$

Hence,

$$p\, e^{-\frac{1}{2}\left(\frac{E\pm\lambda}{\sigma_w}\right)^2} = (1-p)\, e^{-\frac{1}{2}\left(\frac{E\mp\lambda}{\sigma_w}\right)^2}$$

or

$$p\, e^{-\frac{E\lambda}{\sigma_w^2}} = (1-p)\, e^{\frac{E\lambda}{\sigma_w^2}}$$

or

$$e^{\frac{2E\lambda}{\sigma_w^2}} = \frac{p}{1-p}$$

$$\lambda = \frac{\sigma_w^2}{2E} \ln \frac{p}{1-p}$$