Spring 2010
EE 4601: Assignment 3

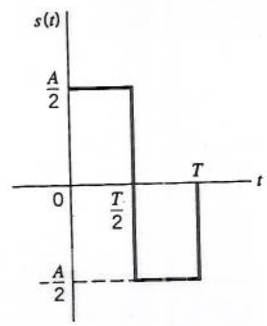
- Date Assigned: February 4, 2010.
- Date Due: February 16, 2010.

1. Consider the signal $s(t)$ shown below

   a) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.

   b) Plot the matched filter output as a function of time.

   c) What is the peak value of the output?



2. Find the matched filter impulse response and peak output signal square to output noise variance of the filter for the following impulse responses. Assume $N_o = 1$, (A, B, C, and D are constants). Also obtain the transfer function of the matched filter in each case.

   a) $g(t) = A \wedge (t-1)$

   b) $g(t) = B\text{rect}\left(\frac{t-2}{4}\right)$

   b) $g(t) = C \wedge (t) u(t)$

   b) $g(t) = D[u(t) - 2u(t-1) + 2u(t-2) - u(t-3)]$

   Note that

$$\wedge(t) = \begin{cases} 1 - |t| , & |t| \leq 1 \\ 0 , & \text{otherwise} \end{cases}$$

$$\text{rect}(t) = \begin{cases} 1 , & |t| \leq 1/2 \\ 0 , & \text{otherwise} \end{cases}$$

3. The figure below shows a pair of orthogonal pulses over the interval $[0, T]$. In this problem we investigate the use of the pulse-pair to study a two-dimensional matched filter.

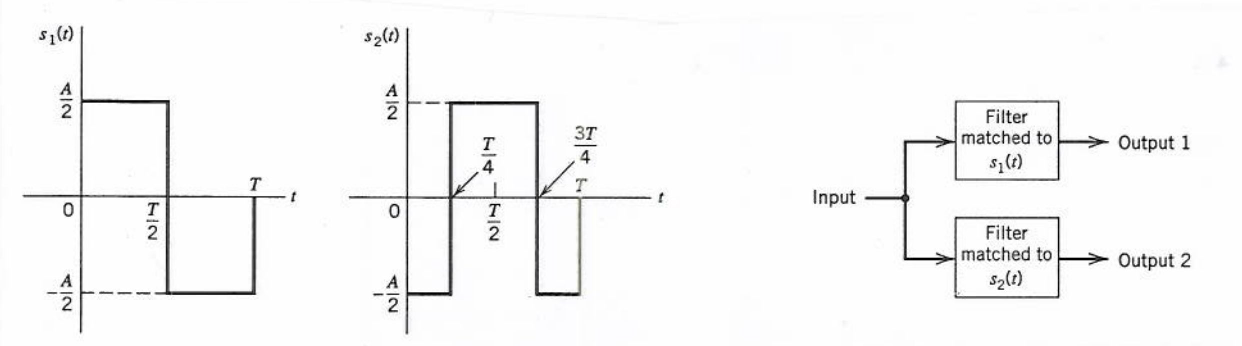
   a) Determine the matched filters for the pulses $s_1(t)$ and $s_2(t)$ considered individually.

   b) Form a two-dimensional matched filter by connecting the two matched filters of Part a) in parallel as shown in the figure below. Hence, demonstrate the following

      i) When the pulse $s_1(t)$ is applied to this two-dimensional filter, the response of the lower matched filter is zero.

      ii) When the pulse $s_2(t)$ is applied to the two-dimensional filter, the response of the upper matched filter is zero.

   c) Generalize the results of your investigation to the case of M orthogonal pulses $s_i(t),\ i = 1, \ldots, M$.



4. Write a MATLAB program to generate the figure on Lecture 9, slide lect9_9 of the lecture notes. This is essentially the same as Figure 3-5 in the text.

5. Assume that

$$s(t) = \begin{cases} \frac{A}{T}t, & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases}$$

is a known signal in the presence of additive white Gaussian noise.

a) Design a matched filter for $s(t)$. Sketch the waveform at the output of the matched filter.

b) Now assume that a correlation detector is used instead. Sketch the waveform at the output of the correlation detector.

6. Consider binary signaling on an additive white Gaussian noise channel. The conditional probability density functions of the matched filter or correlator outputs are

$$f_{y|1}(y|1) = \frac{1}{\sqrt{2\pi}\sigma_w} \exp\{\frac{-(y-E)^2}{2\sigma_w^2}\}, \quad \sigma_w^2 = \frac{N_o E}{2}$$
$$f_{y|0}(y|0) = \frac{1}{\sqrt{2\pi}\sigma_w} \exp\{\frac{-(y+E)^2}{2\sigma_w^2}\}, \quad \sigma_w^2 = \frac{N_o E}{2}$$

Decisions are made such that we choose "1" if $y > \lambda$ and we choose "0" if $y < \lambda$. In Lecture 9, we have seen that the optimum decision threshold (minimizes the bit error probability) is $\lambda = 0$ if $P('1') = P('0') = 1/2$. What is the optimum decision threshold if $P('1') \neq P('0')$? *Hint: Use the theorem of total probability to write down the probability of bit error, P_b, then take the derivative of P_b with respect to λ.*