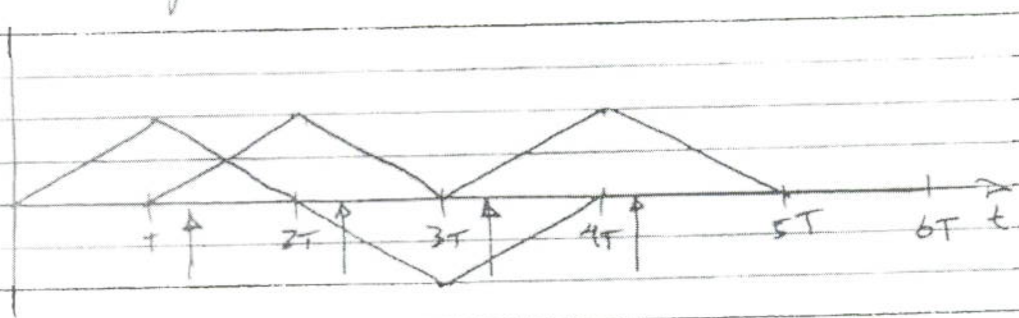
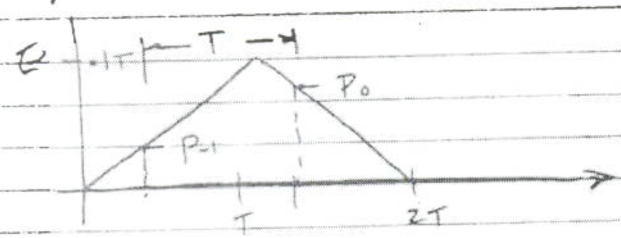
ECE 4601 Homework #4
Solutions.

1. The following diagram shows the typical correlator output



sampling instants at $(1.1)kT$

a) The sampled pulse $p(t)$ is



effective energy is $.9E$ instead of E

b) $p_0 = .9E$

$p_{-1} = .1E$

Loss is $10\log_{10}(.9)$
$= -.46$ dB.

c) At epoch n, the sampled correlator output is

$$y_n = p_0 a_n + p_{-1} a_{n+1} + n_n$$

where $n_n \sim N(0, \frac{N_0 E}{2})$ (see Lecture 9)

Assume wlog that $a_n = +1$

$$P_e = P_{e|a_{n+1}=+1}\, P(a_{n+1}=+1)$$
$$+ P_{e|a_{n-1}=-1}\, P(a_{n+1}=-1)$$

To calculate $P_e|a_{n+1}=+1$ and $P_e|a_{n+1}=-1$ note that

CASE 1: $y_n = 0.9E + 0.1E + n_n$

$= E + n_n$ for $a_{n+1} = +1$

$\Rightarrow P_e|a_{n+1}=+1 = Q\left(\sqrt{\frac{2E}{N_0}}\right)$

CASE 2: $y_n = 0.9E - 0.1E + n_n$

$= 0.8E + n_n$ for $a_{n+1} = -1$

$\Rightarrow P_e|a_{n+1}=-1 = Q\left(\sqrt{\frac{2(0.8)^2E}{N_0}}\right)$

Hence, $P_e = \frac{1}{2} Q\left(\sqrt{\frac{2E}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2(0.8)^2E}{N_0}}\right)$

If no timing error exists

$P_e = Q\left(\sqrt{\frac{2E}{N_0}}\right)$

a) Nyquist bandwidth

$$W = \frac{1}{2T_b} \Rightarrow R = \frac{1}{T_b} = 2400 \text{ bits/s}$$

b) Antipodal Signals

$$P_b = 10^{-7} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\sqrt{\frac{2E_b}{N_0}} \approx 5.2$$ from tables, calculator, etc.

$$\frac{E_b}{N_0} = 13.52 = 11.3 \text{ dB}$$

c) For $\frac{E_b}{N_0} = 13.52$

$$E_b = 13.52 \times N_0$$

$$= 13.52 \times 4.1 \times 10^{-21}$$

$$= 5.54 \times 10^{-20} \text{ J}$$

Received bit energy is $E_b = P_{received} T_b$

$$P_{received} = \frac{E_b}{T_b} = 5.54 \times 10^{-20} \times 2400$$

$$= 1.33 \times 10^{-16} \text{ watts}$$

For 50 Km and 1 dB/km loss, then

There is 50 dB loss

$$P_t = 10^5 \times 1.33 \times 10^{-16} = 1.33 \times 10^{-11} \text{ watts}$$

3. $P_e = \sum_i P_{e/i} P_i$

$= P_{e/i_m=-1/2} \cdot \frac{1}{4} + P_{e/i_m=0} \cdot \frac{1}{2} + P_{e/i_m=1/2} \cdot \frac{1}{4}$

Suppose $a_m = +1$

When $i_m = -1/2$ $y_m = 1/2 + n_m$ $P_{e/i_m=-1/2} = P(y_m < 0)$

$$P_{e/i_m=-1/2} = Q\left(\frac{1/2}{\sigma_n}\right)$$

Likewise $P_{e/i_m=0} = Q\left(\frac{1}{\sigma_n}\right)$

$$P_{e/i_m=1/2} = Q\left(\frac{3/2}{\sigma_n}\right)$$

Hence, $P_e = \frac{1}{4} Q\left(\frac{1}{2\sigma_n}\right) + \frac{1}{2} Q\left(\frac{1}{\sigma_n}\right) + \frac{1}{4}\left(\frac{3}{2\sigma_n}\right)$

4. $H(f) = 1 + \alpha \cos 2\pi f t_0 \qquad |f| < W$

Since $s(t)$ has BW $< W$ we can remove the restriction $|f| < W$

$$H(f) = 1 + \alpha \frac{e^{j2\pi f t_0} + e^{-j2\pi f t_0}}{2}$$

$$= 1 + \frac{\alpha}{2} e^{j2\pi f t_0} + \frac{\alpha}{2} e^{-j2\pi f t_0}$$

$$h(t) = \delta(t) + \frac{\alpha}{2}\delta(t - t_0) + \frac{\alpha}{2}\delta(t + t_0)$$

Hence $y(t) = s(t) * h(t)$

$$= s(t) + \frac{\alpha}{2} s(t - t_0) + \frac{\alpha}{2} s(t + t_0)$$

b) $h(t) = s(-t)$

Let $z(t)$ be the filter output.

$$z(t) = \int_{-\infty}^{\infty} y(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} y(\tau)\, s(-t+\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} s(\tau)\, y(t+\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} s(\tau)\, s(t+\tau)\, d\tau + \frac{\alpha}{2}\int_{-\infty}^{\infty} s(\tau)\, s(t+\tau-t_0)\, d\tau$$

$$+ \frac{\alpha}{2}\int_{-\infty}^{\infty} s(\tau)\, s(t+\tau+t_0)\, d\tau$$

$$z(kT) = \int_{-\infty}^{\infty} s(\tau)\, s(kT+\tau)\, d\tau$$

$$+ \frac{\alpha}{2}\int_{-\infty}^{\infty} s(\tau)\, s(kT+\tau-t_0)\, d\tau + \frac{\alpha}{2}\int_{-\infty}^{\infty} s(\tau)\, s(kT+\tau+t_0)\, d\tau$$

c) when $t_0 = T$

$$z(kT) = \int_{-\infty}^{\infty} s(\tau)\, s(kT+\tau)\, d\tau$$

$$+ \frac{\alpha}{2}\int_{-\infty}^{\infty} s(\tau)\, s((k-1)T+\tau)\, d\tau$$

$$+ \frac{\alpha}{2}\int_{-\infty}^{\infty} s(\tau)\, s((k+1)T+\tau)\, d\tau$$

5. $R_b = 56$ Kb/s

The Nyquist frequency is $W = \frac{R_b}{2} = 28$ KHz

$B_T = W(1+\alpha)$

α	B_T (kHz)
.25	35
.5	42
.75	49
1.0	56