Spring 2010
EE 4601: Assignment 5

- Date Assigned: March 4, 2010.
- Date Due: March 16, 2010.

1. Find an orthonormal basis for the following three signals

$$\begin{aligned} s_1(t) &= u(t) - u(t-2) \\ s_2(t) &= u(t-1) - u(t-3) \\ s_3(t) &= u(t) - u(t-3) \end{aligned}$$

2. Using the gram-Schmidt procedure, obtain an orthonormal basis for the following set of signals

$$\begin{aligned} s_1(t) &= e^{-t}u(t) \\ s_2(t) &= e^{-2t}u(t) \\ s_3(t) &= e^{-3t}u(t) \end{aligned}$$

   Generalize this result to N such signals, i.e.

$$s_n(t) = e^{-nt}u(t)$$

3. An 8-level PAM signal is defined by

$$s_i(t) = A_i \text{rect}\left(\frac{t}{T} - \frac{1}{2}\right)$$

   where $A_i = \pm 1, \pm 3, \pm 5, \ldots, \pm 7$. Formulate the signal constellation for $\{s_i(t)\}$.

4. Assume that the signals in the following two-dimensional signal constellations are equally probable. Find the new signal coordinates in each case so that the average energy is minimized. Plot the signal constellations before and after energy minimization

   (a) $\mathbf{s}_1 = (2,1),\ \mathbf{s}_2 = (2,4),\ \mathbf{s}_3 = (-3,-4)$

   (b) $\mathbf{s}_1 = (0.1),\ \mathbf{s}_2 = (1,3),\ \mathbf{s}_3 = (-5,-4),\ \mathbf{s}_4 = (-1,-4)$

   (c) $\mathbf{s}_1 = (3,2),\ \mathbf{s}_2 = (1,4),\ \mathbf{s}_3 = (-1,-1),\ \mathbf{s}_4 = (-3,-3)$

5. Consider the optimum detection of the sinusoidal signal

$$s(t) = \sin\left(\frac{8\pi t}{T}\right)$$

in additive white Gaussian noise

(a) Determine the correlator output assuming a noiseless input.

(b) Determine the corresponding matched filter output, assume that the filter includes a delay T to make it causal

(c) Hence, show that these two outputs are the same only at the time instant $t = T$.