Spring 2010
EE 4601: Assignment 8

- Date Assigned: April 8, 2010.
- Date Due: April 20, 2010.

1. Differential M-PSK is the M-ary extension of binary DPSK. The absolute excess phase of the modulator at time n, θ_n, is determined recursively by the relation

$$\theta_n = \theta_{n-1} + \left(\frac{2\pi}{M}\right) m_n \quad \text{modulo } 2\pi$$

where θ_{n-1} is the previous excess phase, and $m_n \in \{0, 1, 2, \ldots, M-1\}$ is the modulator input at time n. The probability of symbol error for this M-ary modulation scheme is approximately given by

$$P_M = 2Q\left(\sqrt{\frac{4E}{N_o}} \sin\left(\frac{\pi}{2M}\right)\right), \quad M \geq 4$$

where it is assumed that E/N_o is large.

a Determine the factor by which the transmitted energy per symbol would have to be increased for the differential M-PSK system to attain the same probability of symbol error as coherent M-PSK for $M \geq 4$.

b For $M = 4$, by how many decibels is differential QPSK poorer in performance that coherence QPSK?

2. Consider non-orthogonal coherent binary FSK with the waveforms

$$\begin{aligned} s_1(t) &= A\cos(2\pi f_c t)\ , \quad 0 \leq t \leq T \\ s_2(t) &= A(2\pi(f_c + \Delta_f)t)\ , \quad 0 \leq t \leq T \end{aligned}$$

and assume that $f_c T \gg 1$.

a Show that the correlation coefficient is given by

$$\rho = \frac{1}{E}\int_0^T s_1(t)s_2(t)dt = \text{sinc}(2\Delta_f T)$$

where E is the energy in the waveforms.

b What is the value of Δ_f that minimizes the probably of symbol error? *Hint The answer is not* $\Delta_f = 1/(2T)$.

c For the value of Δ_f obtained in part b), determine the increase in E_b/N_o required so that this coherent FSK system has the same bit error probability as a coherent binary PSK system.

3. Suppose that BPSK signaling is used, such that one of two message waveforms $\tilde{s}_1(t)$ or $\tilde{s}_2(t)$ are transmitted in each bit interval. The received complex envelope is

$$\tilde{r}(t) = g\tilde{s}_i(t) + \tilde{n}(t)$$

where $g = \alpha e^{j\phi}$ and $\tilde{n}(t)$ is additive white Gaussian noise with power spectral density N_o watts/Hz. In each bit period α is an independent identically distributed random variable with probability density function

$$p_\alpha(x) = 0.01\delta(x) + 0.09\delta(x-1) + 0.9\delta(x-2)$$

a What is the average received energy $\bar{E}$ of the *signal* component in the bandpass waveform $r(t)$?

b Determine the probability of bit error, P_b, as a function of

$$\bar{\gamma}_b = \frac{\bar{E}}{N_o}$$

c To what value does P_b approach as $\bar{\gamma}_b \to \infty$?

4. Quadrature multiplexing is to be used to transmit two data streams which differ in bit rate by a factor of 10. What must be the ratio of their amplitudes if the $\gamma_b = \alpha^2 E_b/N_o$ of the demodulated quadrature data streams are to be the same?

5. OFDM systems are known to be resilient to timing errors. Suppose that the OFDM complex envelope

$$\tilde{s}(t) = A\sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi nt}{T}}$$

is sampled at time instants $t = kT_s + \Delta_t$, where Δ_t is a timing offset, to yield the samples

$$R_k = \tilde{s}(kT_s + \Delta_t), k = 0, \ldots, N-1$$

and an N-point FFT is taken on the samples $\{R_k\}_{k=0}^{N-1}$ to yield the coefficients $\{Z_n\}_{n=0}^{N-1}$.

Assume that the timing offset Δ_t lies somewhere in the OFDM guard interval such that all N FFT coefficients belong to the same OFDM block. Determine the FFT coefficients $\{Z_n\}_{n=0}^{N-1}$.