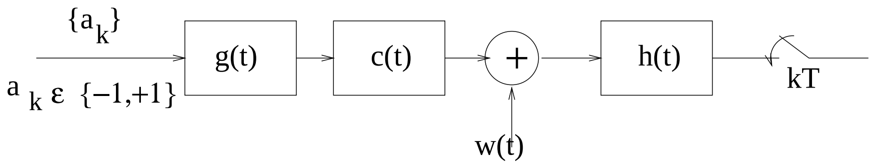
EE4601
Communication Systems

Lecture 12
Partial Response Signals

Objective

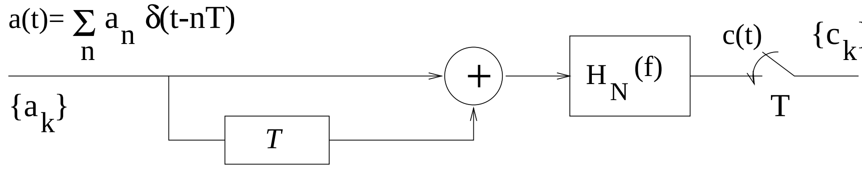
Objective: Signals with a baud rate of 2W symbols/sec in a bandwidth of W Hz with realizable filters.



$$
\begin{aligned}
\text{Assume}\quad c(t) &= \delta(t) \quad \text{(ideal channel)} \\
\text{Then}\quad h(t) &= g(T-t) \\
p(t) &= g(t) * g(-t) \\
P(f) &= |G(f)|^2
\end{aligned}
$$

Partial response signaling

Assume that $P(f)$ has the following form



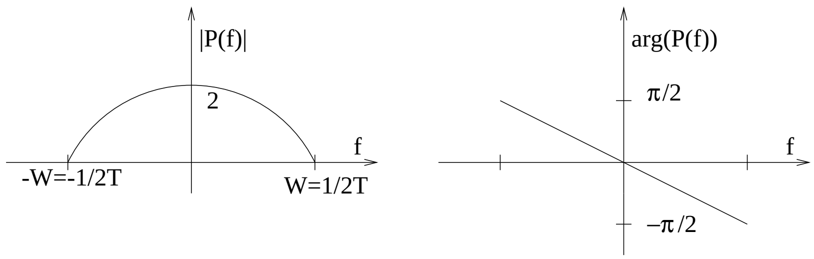
$$H_N(f) = \text{rect}\left(\frac{f}{2W}\right) = \begin{cases} 1 & |f| < W \\ 0 & \text{else} \end{cases}$$

where $W = 1/2T$ i.e., the baud rate is $R = 1/T = 2W$ symbols/sec

$$\begin{aligned} P(f) &= (1 + e^{-j2\pi fT})H_N(f) \\ &= 2e^{-j\pi fT}\left(\frac{e^{j\pi fT} + e^{-j\pi fT}}{2}\right)H_N(f) \\ &= 2\cos(\pi fT)e^{-j\pi fT}H_N(f) \\ &= 2\cos(\pi fT)e^{-j\pi fT}\text{rect}\left(\frac{f}{2W}\right) \end{aligned}$$

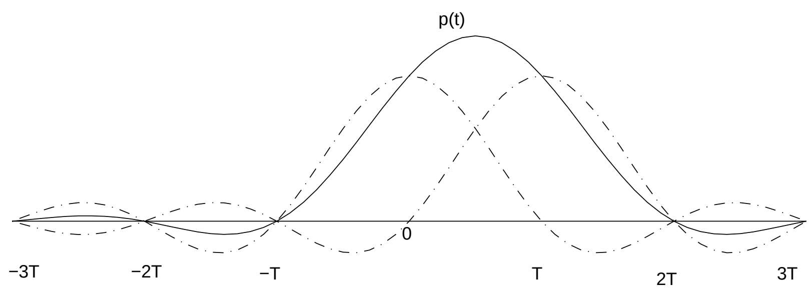
Duobinary

$$P(f) = \begin{cases} 2\cos(\pi f T) e^{-j\pi f T} & |f| < W \\ 0 & \text{else} \end{cases}$$

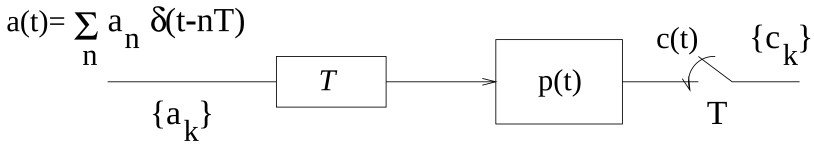


To get $p(t)$ we write

$$\left\{ P(f) = H_N(f) + H_N(f) e^{j2\pi f T} \right\} \leftrightarrow \left\{ p(t) = \text{sinc}\left(\frac{t}{T}\right) + \text{sinc}\left(\frac{t-T}{T}\right) \right\}$$



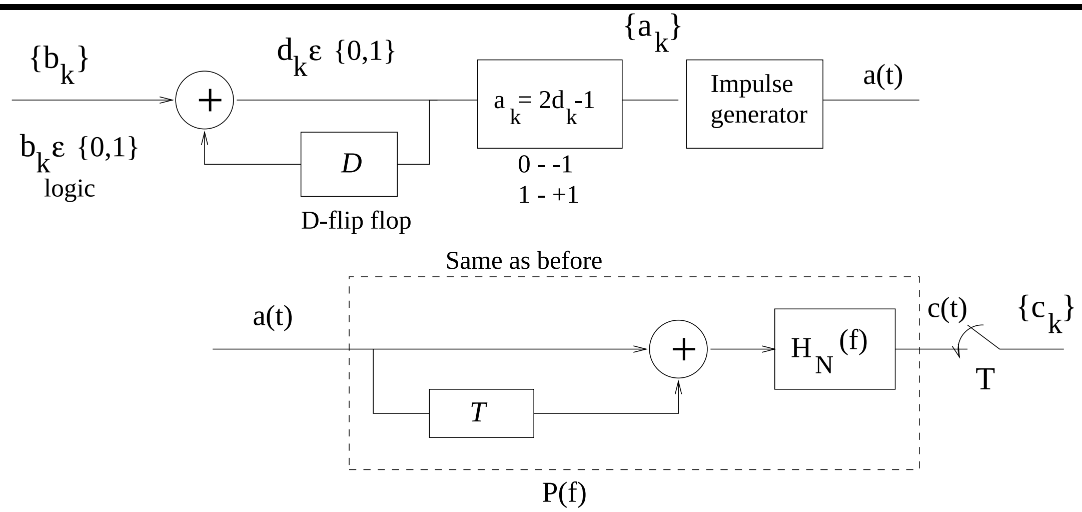
Duobinary



$$
\begin{aligned}
c(t) &= \sum_n a_n p(t - nT) \\
c_k = c(kT) &= \sum_n a_n p((k-n)T) = \sum_n a_n p_{k-n} \\
\text{But} \quad p_j = p(jT) &= \begin{cases} 1 & j = 0, 1 \\ 0 & j \neq 0, 1 \end{cases} \\
\text{Therefore,} \quad c_k &= a_k + a_{k-1}
\end{aligned}
$$

Since $a_k \in \{-1, +1\}$ $c_k \in \{-2, 0, 2\}$ (3-level)
We can recover $\{a_k\}$ from $\{c_k\}$ by $a_k = c_k - a_{k-1}$ assuming an initial value, e.g. $a_0 = -1$ or $+1$. This is called <u>decision feedback</u> detection.
Problem : Errors due to noise propagate, i.e., $\hat{a_k} = c_k - \hat{a_{k-1}}$.
If $\hat{a_{k-1}}$ is in error then $\hat{a_k}$ is likely to be in error.

Precoding



$$d_k = b_k \oplus d_{k-1} = b_k + d_{k-1} (\text{mod } 2)$$

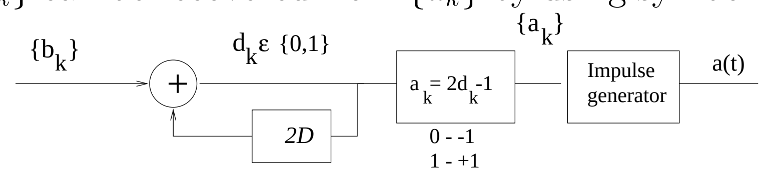
Example:

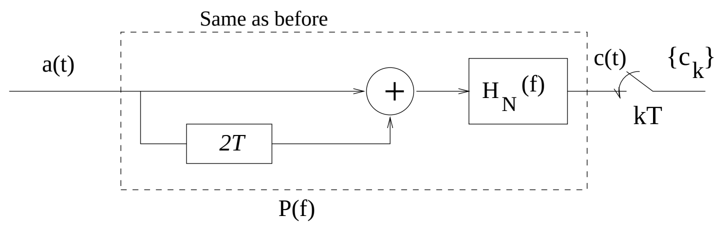
$\{b_k\}$		0	0	1	1	1	0	1	0	0	0
$\{d_k\}$	1	1	1	0	1	0	0	1	1	1	1
$\{a_k\}$	$+1$	$+1$	$+1$	-1	$+1$	-1	-1	$+1$	$+1$	$+1$	$+1$
$\{c_k\}$		$+2$	$+2$	0	0	0	-2	0	$+2$	$+2$	$+2$

Modified Duobinary

Note that $c_k = \begin{cases} \pm 2 & \text{if} \quad b_k = 0 \\ 0 & \text{if} \quad b_k = 1 \end{cases}$

Therefore $\{b_k\}$ can be recovered from $\{a_k\}$ by using symbol by symbol detection.

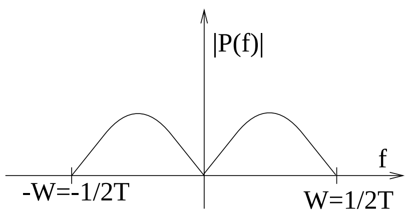


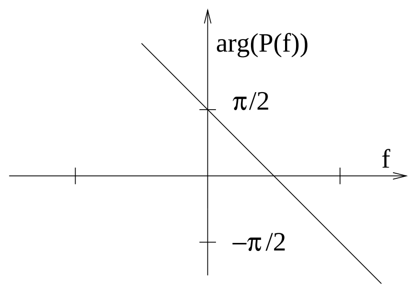


$a(t) = \sum_n a_n \delta(t - nT)$, $d_k = b_k \oplus d_{k-2}$, $c_k = a_k - a_{k-2}$.

Modified Duobinary

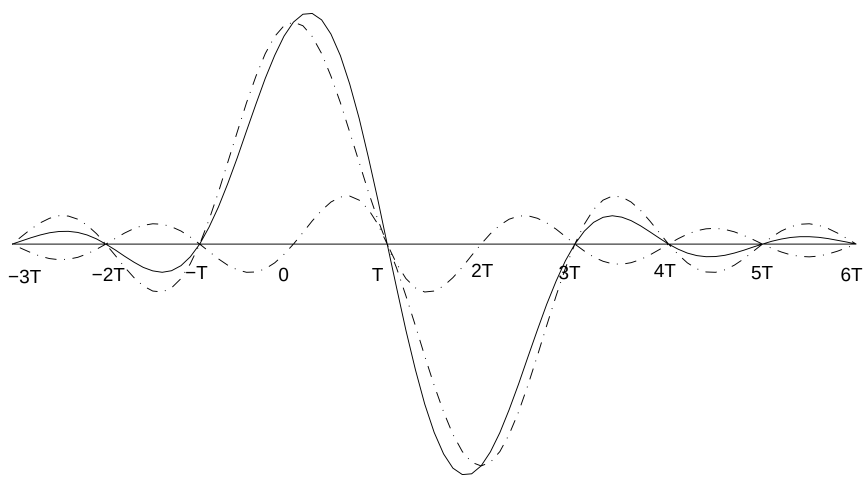
$$
\begin{aligned}
P(f) &= (1 - e^{-j4\pi fT}) H_N(f) \\
&= j2e^{-j2\pi fT} \left(\frac{e^{j2\pi fT} - e^{-j2\pi fT}}{j2} \right) H_N(f) \\
&= j2H_N(f) \sin 2\pi fT e^{-j2\pi fT} \\
&= \begin{cases} 2 \sin 2\pi fT e^{j(\pi/2 - 2\pi fT)} & |f| < 1/2T \\ 0 & |f| > 1/2T \end{cases}
\end{aligned}
$$





Modified Duobinary Pulse

$$P(f) = H_N(f) - e^{-j4\pi fT} H_N(f)$$
$$= \text{sinc}(t/T) - \text{sinc}((t-2T)/T)$$



Modified Duobinary with Precoding

Example:

$\{b_k\}$			0	0	1	1	1	0	1	0	0	0
$\{d_k\}$	1	1	1	1	0	0	1	0	0	0	0	0
$\{a_k\}$	$+1$	$+1$	$+1$	$+1$	-1	-1	$+1$	-1	-1	-1	-1	-1
$\{c_k\}$			0	0	-2	-2	2	0	-2	0	0	0

Note: $c_k = \begin{cases} \pm 2 & \text{if} \quad b_k = 1 \\ 0 & \text{if} \quad b_k = 0 \end{cases}$